CP and MIP Methods for Ship Scheduling with Time-Varying Draft

Elena Kelareva[1]
Sebastian Brand[2]
Philip Kilby[1]
Sylvie Thiébaux[1]
Mark Wallace[3]

1. NICTA / ANU
2. NICTA / University of Melbourne
3. NICTA / Monash University


Photograph: Marj Kibby

Outline

Problem Introduction

- Draft Restrictions in Ship Scheduling
- Maximising Cargo Throughput at a Bulk Export Port

Modelling the Problem

- Constraint Programming Model
- Tug Constraints
- CP vs MIP

Improving the CP Model

- Sequence Variables
- 2D arrays
- Sorted Inputs

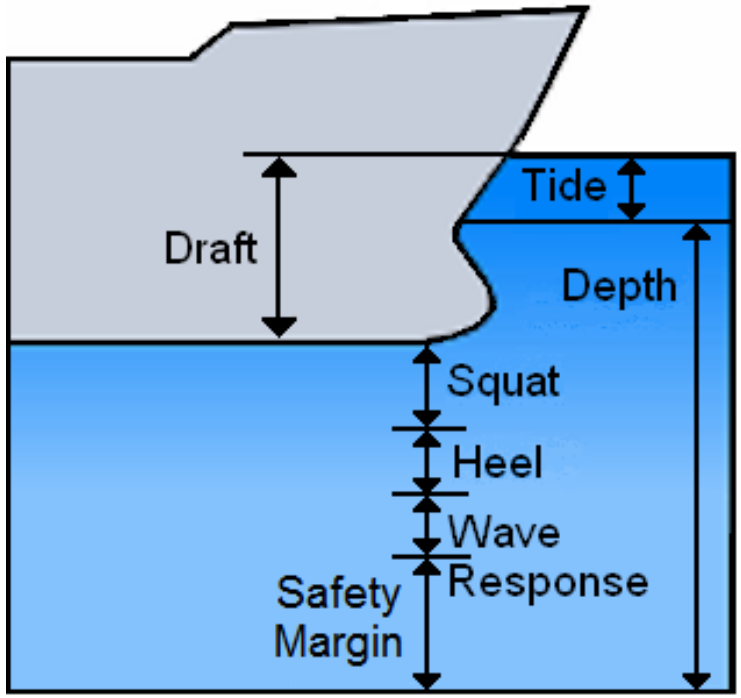
Ship Scheduling Background

NICTA

What is ship scheduling?

- Choose sailing times and amounts of cargo
- Maximise profit / minimise cost
- Obey port safety rules (eg. Draft)

Time-Varying Draft Restrictions



- Ports have safety restrictions on draft
- Draft increases as more cargo loaded
- Existing approaches use constant draft restrictions
- Time-varying draft → increased accuracy → more cargo → more profit

Can sail if:

Tide + Depth – Draft – Squat – Heel – WR ≥ Safety Margin

[O'Brien 2002]

Maximising Cargo Throughput at a Bulk Export Port

Outbound ships full; inbound empty

Narrow channel → sequence-dependent separation times

Objective: maximise cargo for a set of ships

Commercial System: Input Ship Parameters

NICTA

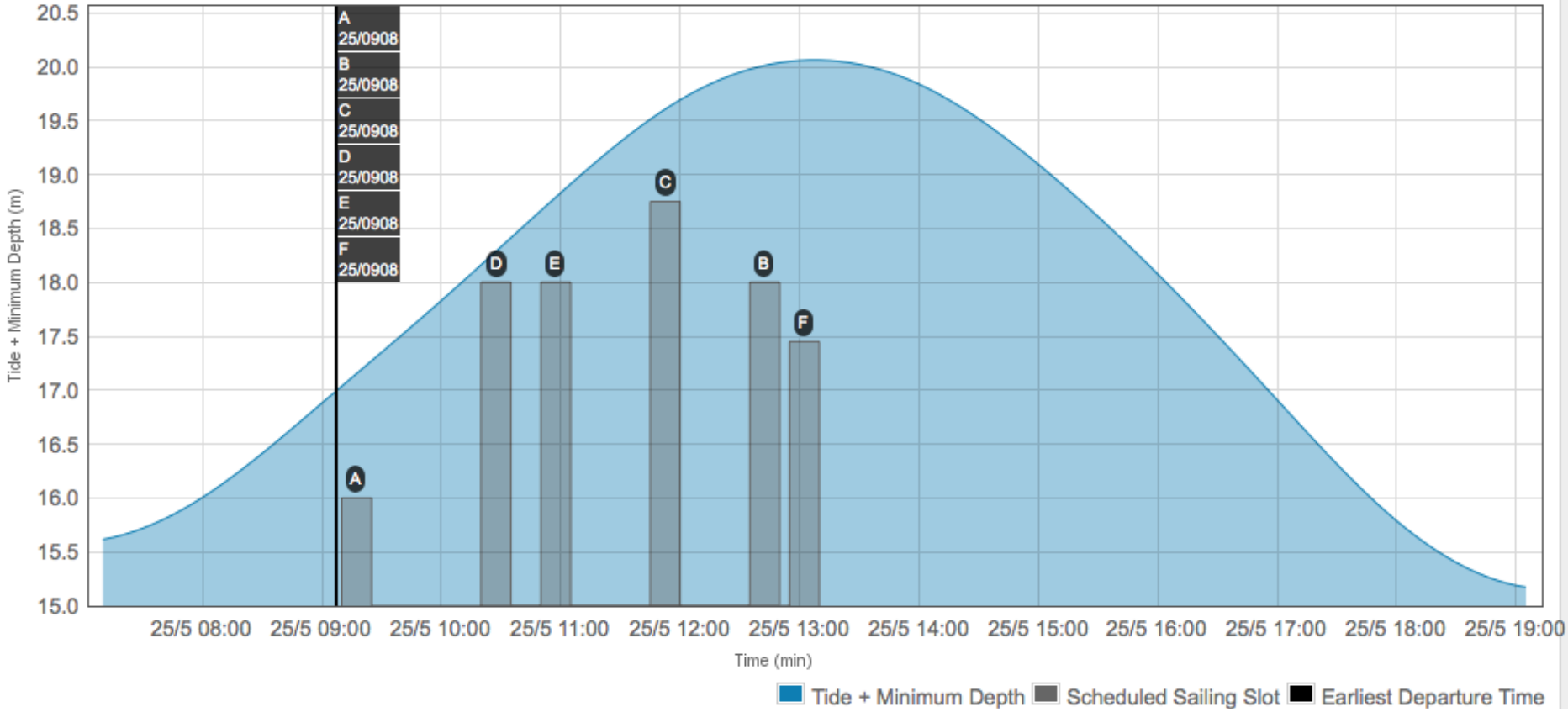
Vessel Name	CAMELLIA 50m 300m
Gmf (m)	
KG (m)	

Earliest Departure	25May2012 0908
Requested Draft (m)	19
Min Draft (m)	18
Max Draft (m)	19
Lock Draft	☐
Lock Time	☐

Priority	3
Sail	EARLY
Berth	NPB
Destination	B2
Pilotage	SEA
Tugs	4
NTugs Early Turnaround	1

Commercial System: Output

Vessel Beam, LBP	Sailing Draft	Sailing Slot Open Time	Sailing Slot Close Time
(A) AMIRA (32.26m, 217m)	16.00m	25May2012 0910	25May2012 0925
(B) BEGONIA (45m, 280.8m)	18.00m	25May2012 1235	25May2012 1250
(C) CAMELLIA (50m, 300m)	**18.75m**	25May2012 1145	25May2012 1200
(D) DIONE (45m, 283m)	18.00m	25May2012 1020	25May2012 1035
(E) EURYDICE D (42.5m, 267.6m)	18.00m	25May2012 1050	25May2012 1105
(F) FIRST JUPITER (45m, 277m)	**17.45m**	25May2012 1255	25May2012 1310



CP Model Features

Optimality

- 1cm extra draft = 130t extra cargo
- Even a small reduction in quality is undesirable
- Aim to find an optimal schedule

Oversubscribed problem

- Undersubscribed in general; oversubscribed at high tide

Time-indexed formulation

- Draft changes over time
- Modelled with 5-minute time increments

Sequence-dependent constraints

- Separation times between ships depend on ship order
- Tug constraints depend on ship order

Basic CP Model

Earliest Departure Time:

$$s(v)=1 \rightarrow T(v) \geq E(v) \forall v \in V$$

Berth Availability:

$$s(B_i(b))=1 \rightarrow$$
$$s(B_o(b))=1 \wedge T(B_o(b)) \leq T(B_i(b))-d(b) \forall b \in B$$

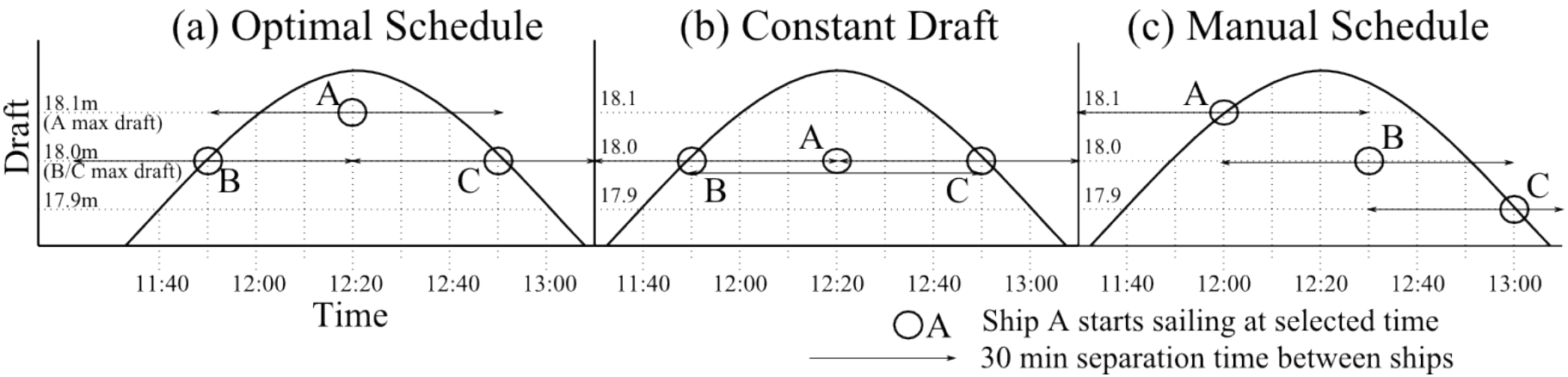
Separation Time:

$$s(v_i)=1 \wedge s(v_j)=1 \rightarrow$$
$$T(v_j)-T(v_i) \geq ST(v_i, v_j) \vee T(v_i)-T(v_j) \geq ST(v_j, v_i) \forall v_i, v_j \in V$$

Objective Function:

$$\text{maximise} \sum_{v \in V} s(v).C(v).D(v, T(v))$$

Comparison Against Existing Approaches



Constant draft restrictions and manual schedules

- 10cm less total draft
- 1300 tonnes less cargo
- $221,000 less iron ore

Larger problems in paper

- 120cm more draft per set of ships than constant draft
- 15.8cm more draft than manual scheduling

Tug Constraints

Tug Constraints: First Attempts

NICTA

Sequence-dependent turnaround times

- Tug constraints scale badly

Allocating tugs to ships: too slow!

Counting busy tugs

- First ensure feasibility, then assign tugs
- Faster than allocating tugs
- Turnaround times are still sequence-dependent
- Still need to track tug origins and destinations: slow

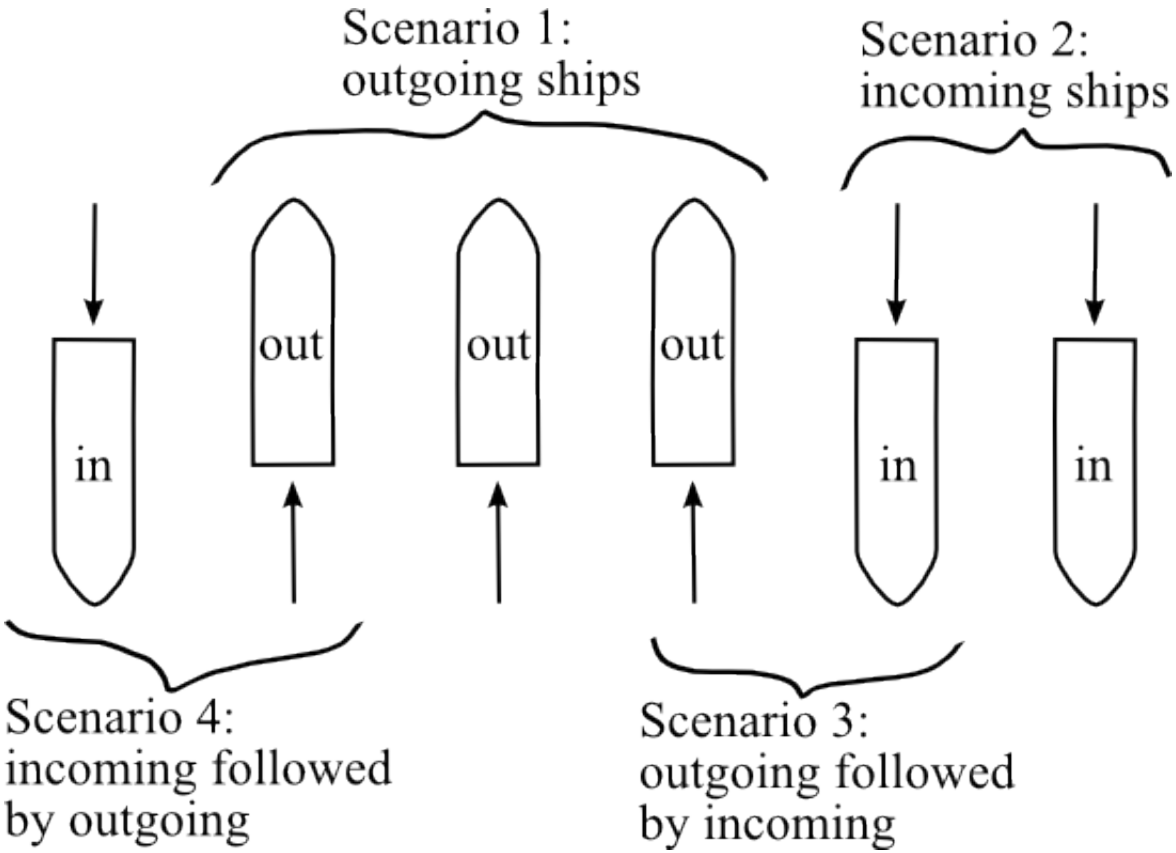
Tug Constraints: Faster Model

2 Key Assumptions:

- Ships cannot pass each other in the channel
- Berths are close enough together that travel time from berth to sea or vice versa is independent of berth location

Incoming and outgoing tugs can be tracked separately

Tug Constraints: 4 Scenarios



Constraint Programming Model: Tugs

Scenarios 1 and 2: One-Directional Sequence of Ships

$$s(v)=1 \wedge t \geq T(v) \wedge t < T(v)+r(v,g) \rightarrow$$
$$U(v,t,g)=H(v,g), \forall v \in V, t \in [1, T_{max}], g \in [1, G_{max}]$$

$$s(v)=0 \vee t < T(v) \vee t \geq T(v)+r(v,g) \rightarrow$$
$$U(v,t,g)=0, \forall v \in V, t \in [1, T_{max}], g \in [1, G_{max}]$$

Scenario 4: Incoming Followed by Outgoing

$$L(v_i,t) \Leftrightarrow \exists v_o \in O \; s.t.$$
$$t=T(v_o) \wedge T(v_i) \leq T(v_o) \wedge T(v_i)+max(r(v_i,g), g \in [1, G(v_i)])+X(v_i,v_o) > T(v_o),$$
$$\forall v_i \in I, t \in [1, T_{max}]$$

$$x(v_i,t)=bool2int(L(v_i,t)). \sum_{g \in [1, G(v_i)]} H(v_i,g), \forall v_i \in I, t \in [1, T_{max}]$$

Scenario 3: Outgoing Followed by Incoming

$$x(v_o,t)=0, \forall v_o \in O, t \in [1, T_{max}]$$

Tug Availability Constraints

$$\sum_{v \in I} \sum_{g \in G(v)} U(v,t,g) \leq U_{max}, \forall t \in [1, T_{max}]$$

$$\sum_{v_o \in O} \sum_{g \in G(v_o)} U(v_o,t,g) + \sum_{v_i \in I} X(v_i,t) \leq U_{max}, \forall t \in [1, T_{max}]$$

Mixed Integer Programming Model

Time Slot Definition:

$$T(v) = \sum_{t \in [1, T_{max}]} s(v,t).t \; \forall v \in V$$

Ship Uniqueness Constraints:

$$\sum_{t \in [1, T_{max}]} s(v,t) \leq 1 \; \forall v \in V$$

Earliest Departure Time:

$$T(v) \geq E(v) \; \forall v \in V$$

Berth Availability:

$$T(B_o(b)) \leq T(B_i(b)) - d(b) \wedge \sum_{t \in [1, T_{max}]} s(B_o(b), t) \geq \sum_{t \in [1, T_{max}]} s(B_i(b), t) \; \forall b \in B$$

Separation Time:

$$s(v_i, t) + \sum_{t' \in [t, min(T_{max}, t + ST(v_i, v_j))]} s(v_j, t') \leq 1 \wedge s(v_j, t) + \sum_{t' \in [t, min(T_{max}, t + ST(v_j, v_i))]} s(v_i, t') \leq 1$$

$$\forall v_i, v_j \in V, t \in [1, T_{max}]$$

MIP Model: Tugs

Scenarios 1 and 2: One-Directional Sequence of Ships

$$U(v,t,g)=H(v,g).\sum_{t'\in[min(1,t-r(v,g)+1),t]} s(v,t'),$$

$$\forall v\in V, t\in[1,T_{max}], g\in[1,G_{max}]$$

Scenario 4: Incoming Followed by Outgoing

$$x(v_i,t)=L(v,t).\sum_{g\in[1,G(v_i)]} H(v_i,g), \forall v_i\in I, t\in[1,T_{max}]$$

$$L(v_i,t)\geq s(v_o,t)+\sum_{t'\in[max(1,t-t_{range}),t]} s(v_i,t)-1, \forall v_i\in I, v_o\in O, t\in[1,T_{max}]$$

where $t_{range}=t-X(v_i,v_o)-max\{r(v_i,g), g\in[1,G(v_i)]\}+1$

Scenario 3: Outgoing Followed by Incoming
(Identical to CP model)

Tug Availability Constraints
(Identical to CP model)

Computational Results

NICTA

Model implemented in MiniZinc

- G12 finite domain CP solver
- OSI CBC MIP solver

Problems of varying size and constrainedness

- Problem sizes: 4-10 ships (realistic for bulk export port)
- “one-way” vs “mixed” ship direction
- “narrow” vs “wide” windows at maximum draft
    - Schedule is more or less constrained at peak of tide
- 5-minute cutoff time

Search strategies

- CP: *Ship Scheduling with Time-Varying Draft Restrictions* E. Kelareva, P. Kilby, S. Thiėbaux and M. Wallace (2012) Odysseus Workshop on Freight Transportation and Logistics.
- MIP: solver chooses search strategy

Computational Results

Problem	CP	MIP
MW	10 (3.17)	8 (39.1)
OW	10 (0.45)	9 (41.8)
MN	10 (7.99)	8 (11.4)
ON	8 (42.5)	7 (11.4)
MWT	10 (115)	6 (180)
OWT	8 (1.76)	8 (273)
MNT	8 (4.06)	6 (126)
ONT	6 (10.3)	5 (7.66)

Table 1: Comparison of MIP vs CP.

Largest problem solved within 5 min cutoff (calc time in brackets)

CP faster than MIP

MIP may do better with other solvers or search strategies

Improving the CP Model

NICTA

Sequence variables

- Additional variables specifying ordering between ships
- Search on sequence variables first

Converting multidimensional array lookups to 1D arrays

- Objective: $\Sigma\ D(v, T(v))$
- Replace with $D'_v\ (T(v))$ where D'_v is the projection of D on v
- More efficiently implemented in G12 FD solver

Sorting ships for improved search efficiency

- Order by decreasing max draft
- Assign times to biggest ships first

CP Model Improvements: Results

Problem	OLD	SEQVARS	ONEDIM	SORT	ONEDIMSORT
MW	10 (3.17)	10 (2.09)	**10 (0.22)**	10 (1.00)	10 (0.56)
OW	10 (0.45)	10 (2.09)	10 (2.62)	10 (0.56)	**10 (0.34)**
MN	10 (7.99)	9 (298)	**10 (0.22)**	10 (8.30)	10 (0.56)
ON	8 (42.5)	8 (57.9)	10 (190)	8 (37.1)	**10 (157)**
MWT	10 (115)	**10 (14.1)**	10 (14.3)	10 (124)	10 (19.4)
OWT	8 (1.76)	8 (14.4)	9 (124)	8 (2.09)	**9 (100)**
MNT	8 (4.06)	8 (137)	10 (166)	8 (4.16)	**10 (121)**
ONT	6 (10.3)	6(13.0)	9 (228)	6 (7.75)	**9 (211)**

Table 2: Comparison of modified CP models.

Largest problem solved within 5 min cutoff (calc time in brackets)

Green = faster than original CP model; yellow = slower

OneDim gives a larger improvement than Sort

SeqVars alone: slightly slower in general

1D array conversion could be automated: MiniZinc update?

CP Model Improvements: Search Strategies

	OneDimSort		OneDim		Sort	
Problem	Draft	SeqVars	Draft	SeqVars	Draft	SeqVars
MW	**10 (0.45)**	10 (0.56)	**10 (0.22)**	**10 (0.22)**	**10 (1.00)**	8 (179)
OW	**10 (0.33)**	**10 (0.34)**	**10 (2.40)**	10 (2.62)	**10 (0.56)**	8 (136)
MN	**10 (0.33)**	10 (0.56)	**10 (0.22)**	**10 (0.22)**	**10 (8.30)**	8 (54.5)
ON	8 (22.8)	**10 (157)**	8 (18.4)	**10 (190)**	**8 (37.1)**	9 (204)
MWT	10 (50.7)	**10 (19.4)**	10 (48.8)	**10 (14.3)**	**10 (124)**	7 (9.83)
OWT	8 (0.89)	**9 (100)**	8 (1.43)	**9 (124)**	**8 (2.09)**	8 (257)
MNT	9 (40.1)	**10 (121)**	9 (27.8)	**10 (166)**	**8 (4.16)**	7 (21.9)
ONT	7 (119)	**9 (211)**	7 (98.3)	**9 (228)**	**6 (7.75)**	8 (282)

Table 3: Searching on draft vs sequence variables for three modified CP models.

- Original CP model: draft >> time
- Sequence variables alone / sorted inputs:
  draft > seq. vars
- With 1D Arrays: seq. vars > draft
- Seq. var. constraints propagate better with 1D arrays?

Conclusions

Introduced time-varying draft to ship scheduling

- Novel problem: optimising cargo throughput at bulk export port
- More cargo than fixed-draft and manual approaches
- Tug constraints solved by splitting into scenarios

CP faster than MIP

3 improvements to CP model

- Sequence variables
- 1D arrays
- Sorted inputs

1D arrays may have implications for other models and MiniZinc improvements

Future Work

Other CP solvers (Chuffed, Gecode, others)

Extension to larger ship routing or mining supply chain problems

- Multiple ports
- Longer time horizon
- Decomposition approach may be needed

MIP model may do better with other solvers/search strategies

Thank you!