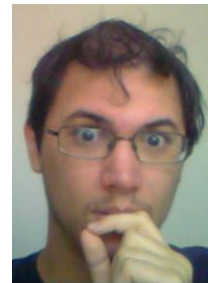


Incremental ARA*: An Anytime Incremental Search Algorithm For Moving Target Search



Xiaoxun Sun

William Yeoh

Tansel Uras

Sven Koenig

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Singapore Management University

Moving Target Search



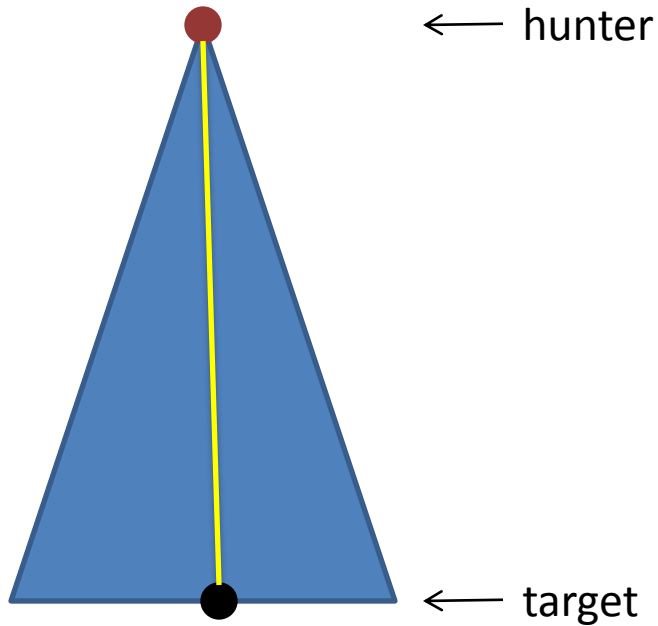
- Assumptions
 - The hunter knows the terrain.
 - The hunter knows its own cell.
 - The hunter knows the cell of the target.

Moving Target Search

- Offline search
 - e.g. minimax search (Reverse Minimax A*)
- Online search
 - e.g. repeated deterministic searches
 - The hunter finds a short path to the target and moves along the path.
 - Whenever the target moves off the path, the hunter repeats the process.

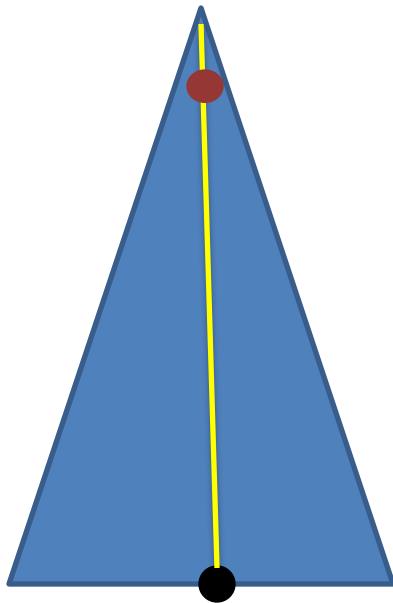
A^* [Hart, Nilsson, Raphael, 1968]

- The hunter uses A^* (with consistent h-values).



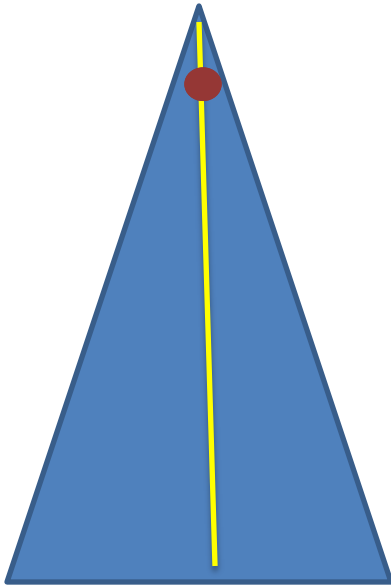
A^* [Hart, Nilsson, Raphael, 1968]

- The hunter uses A^* .



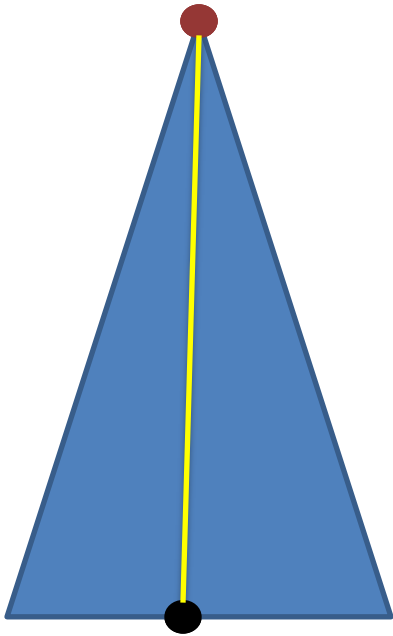
A^* [Hart, Nilsson, Raphael, 1968]

- The hunter uses A^* .



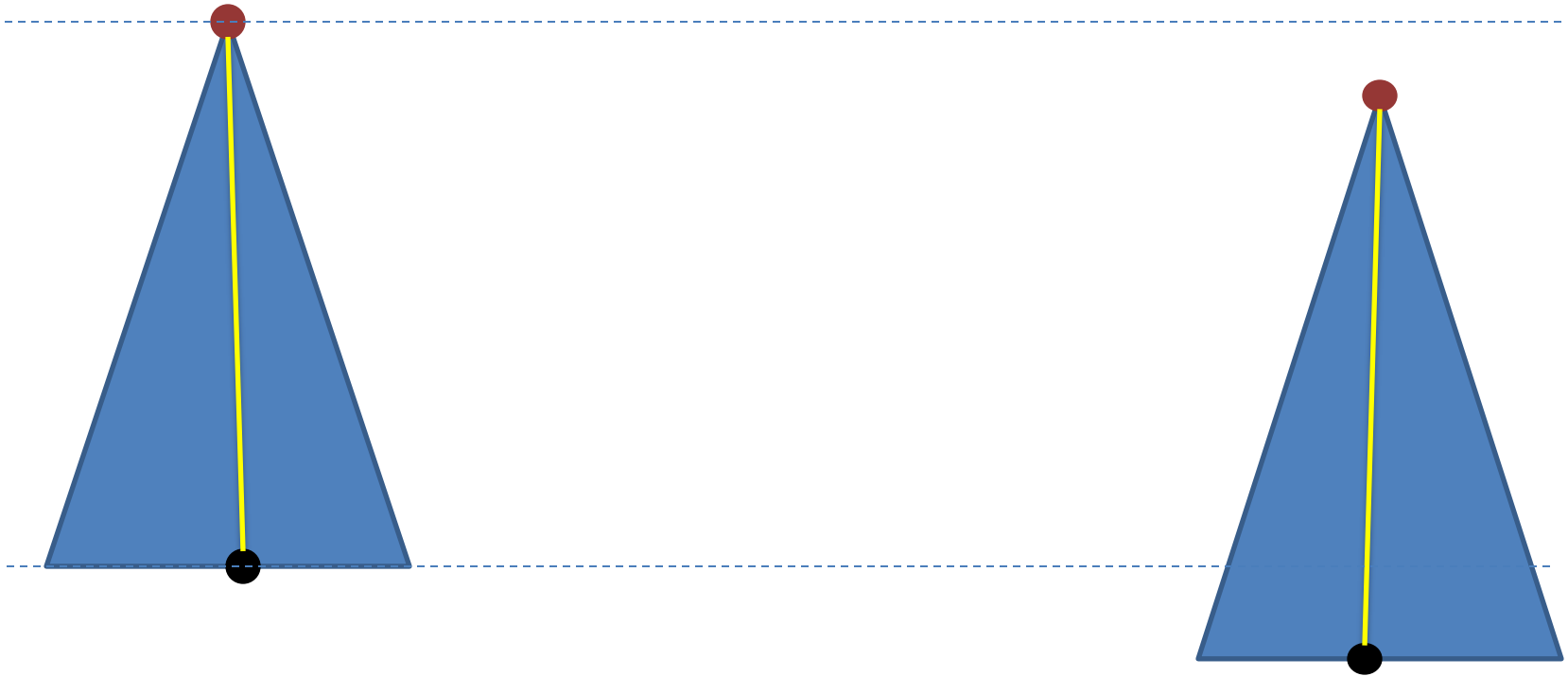
A^* [Hart, Nilsson, Raphael, 1968]

- The hunter uses A^* .

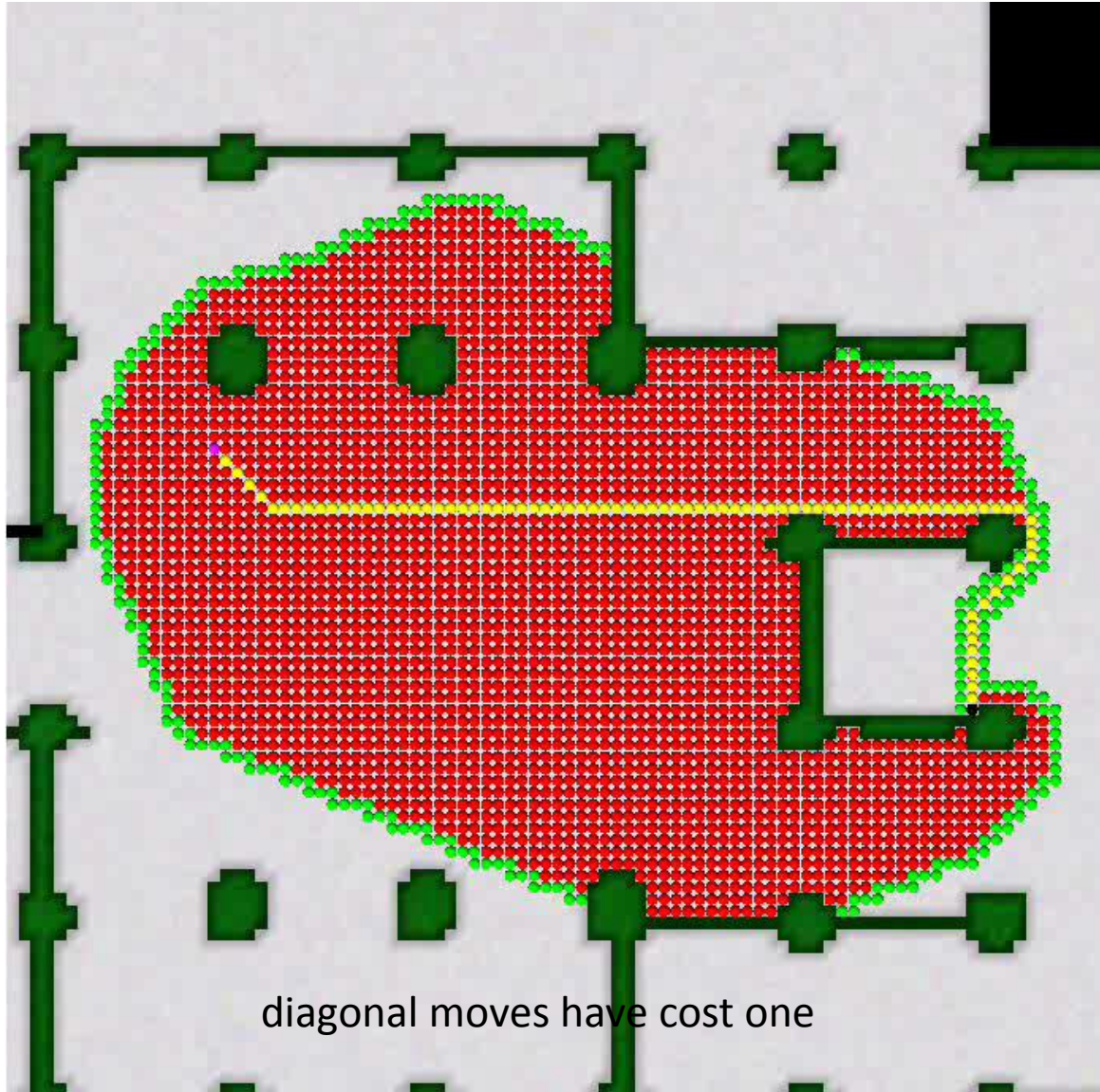


A^* [Hart, Nilsson, Raphael, 1968]

- The hunter uses A^* .



A^* [Hart, Nilsson, Raphael, 1968]



A^* [Hart, Nilsson, Raphael, 1968]



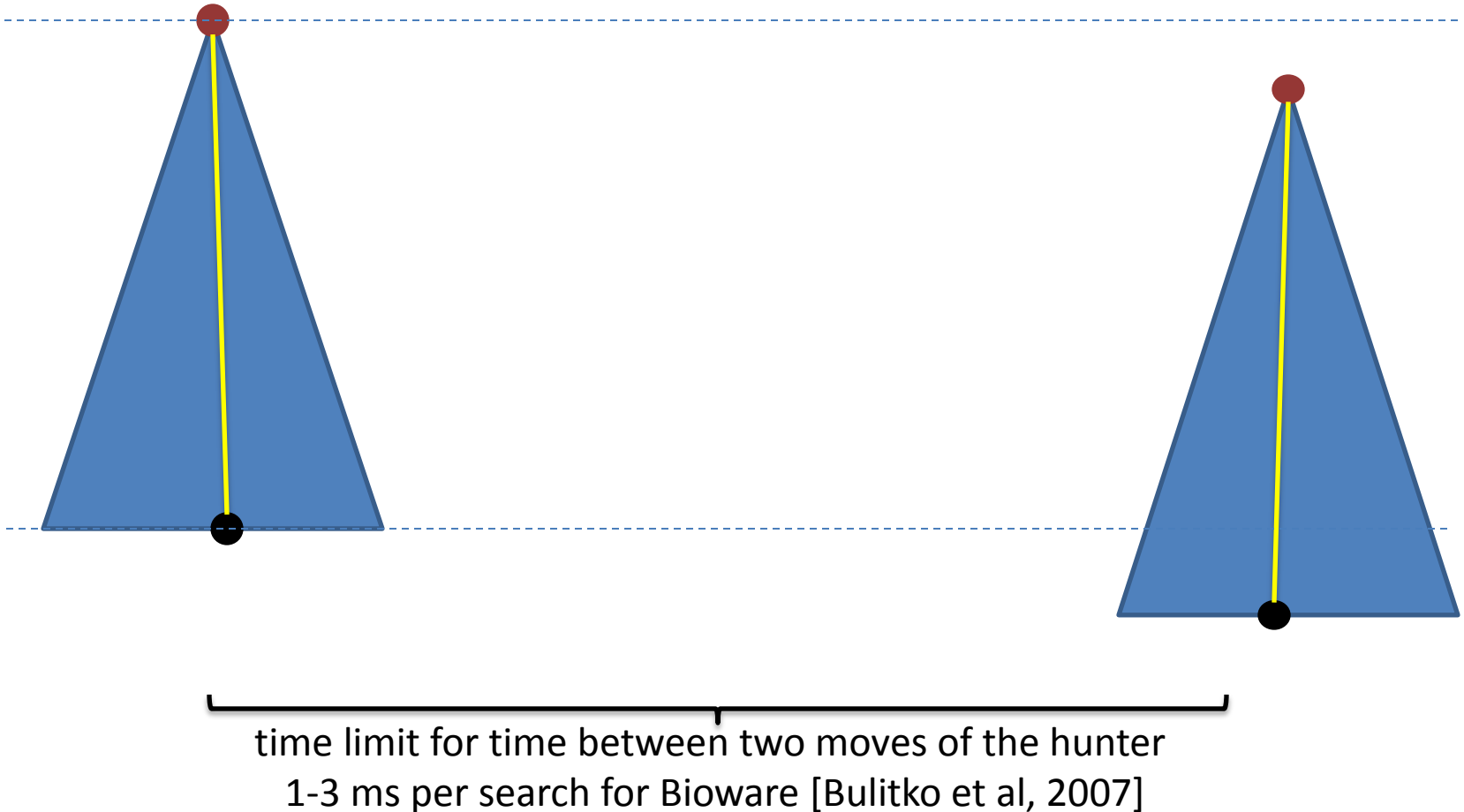
A^* [Hart, Nilsson, Raphael, 1968]

1 move

Small (but soft) **time limit** for time between two moves of the hunter
1-3 ms per search for Bioware [Bulitko et al, 2007]

A^* [Hart, Nilsson, Raphael, 1968]

- The hunter uses A^* .



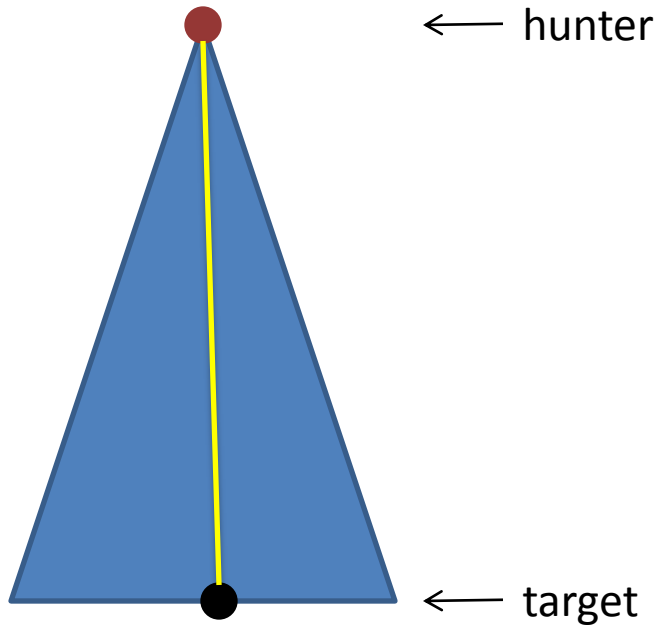
FRA*

[Sun, Yeoh, Koenig, 2009]

- Idea 1:
 - Reduce the runtime of the A* search by using incremental A* search

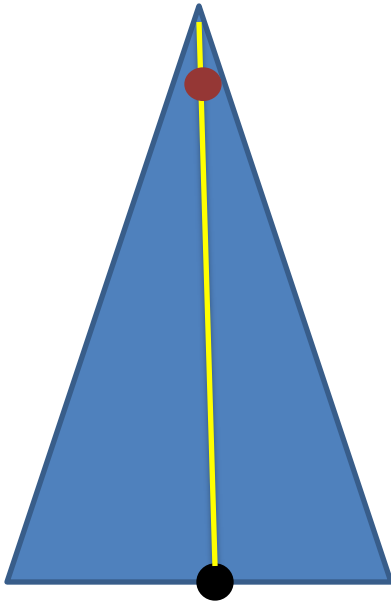
FRA*

- The hunter uses A^* .



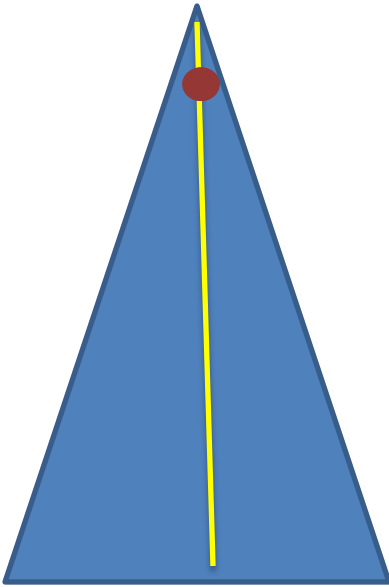
FRA*

- The hunter uses A^* .



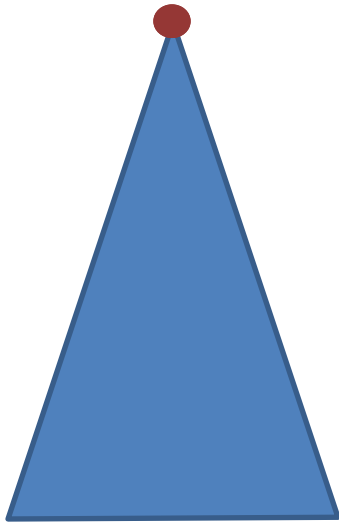
FRA*

- The hunter uses A^* .



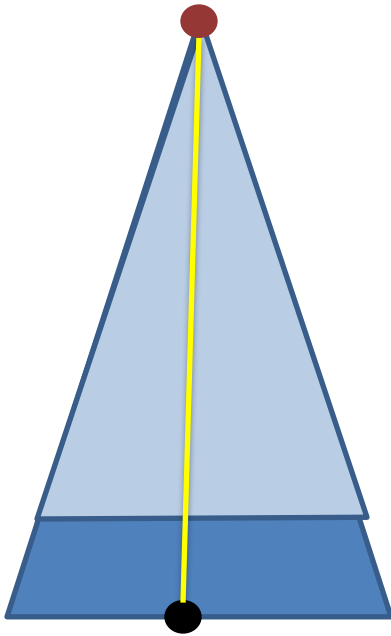
FRA*

- The hunter uses A^* .

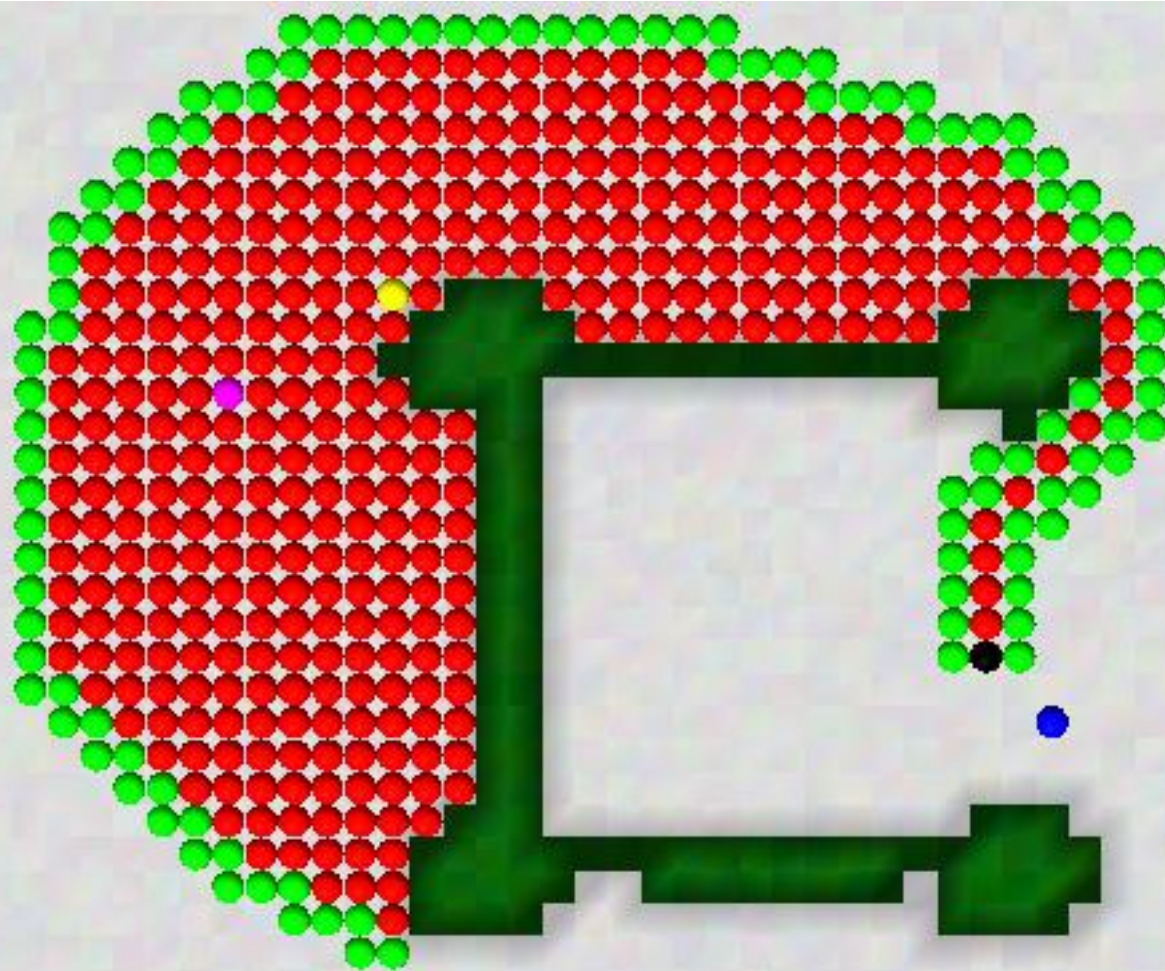


FRA*

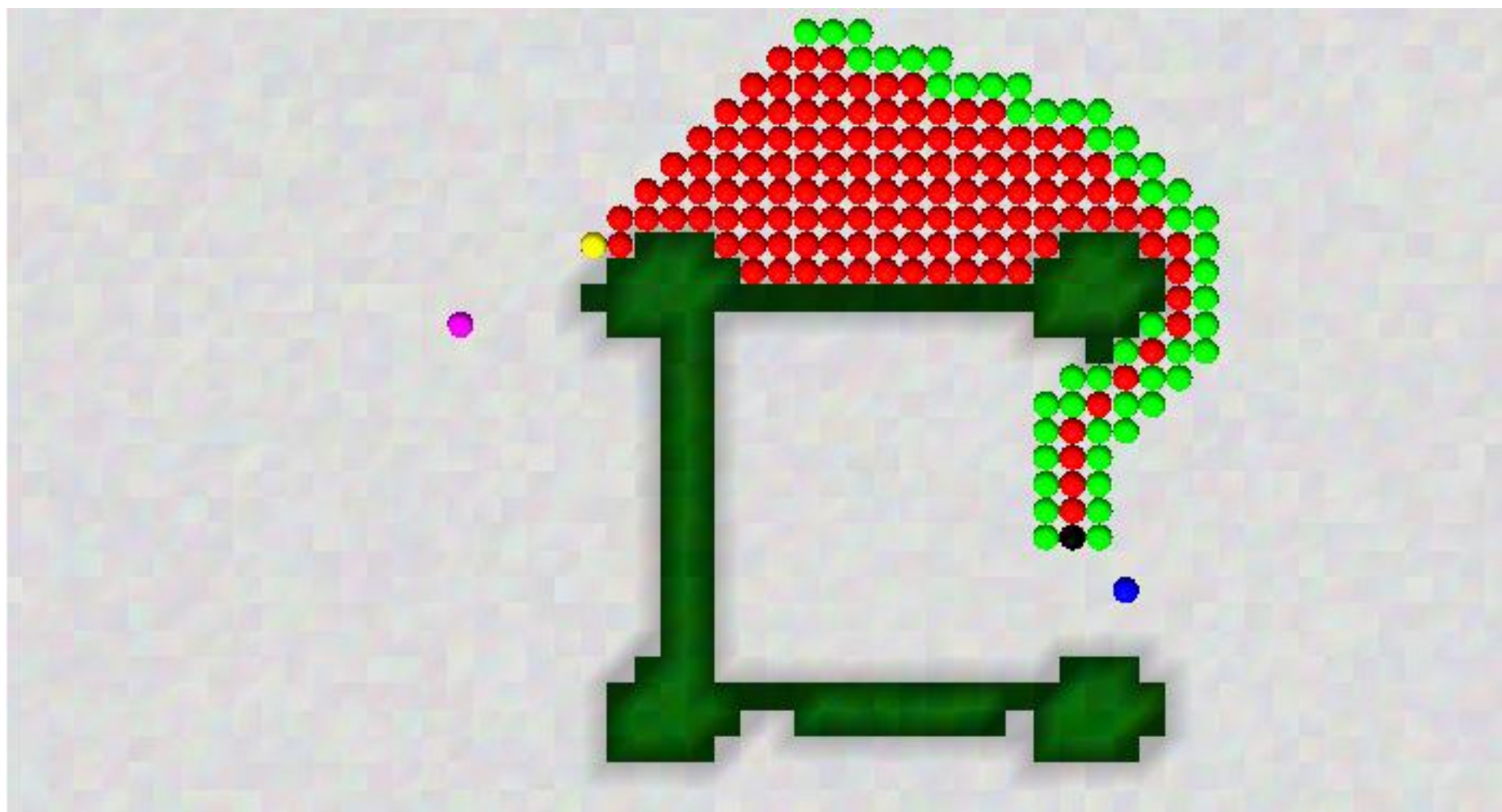
- The hunter uses A^* .



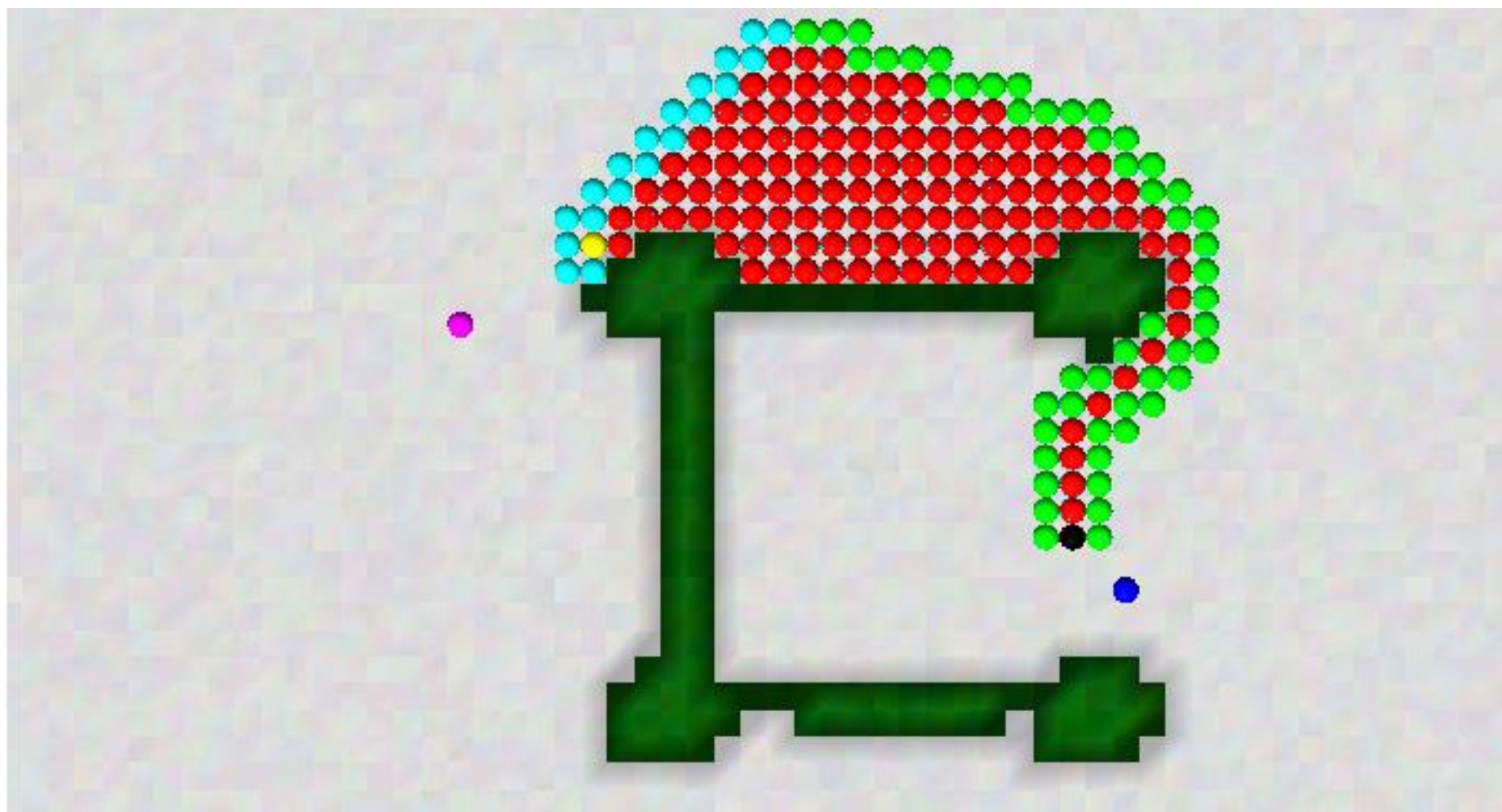
FRA*



FRA*

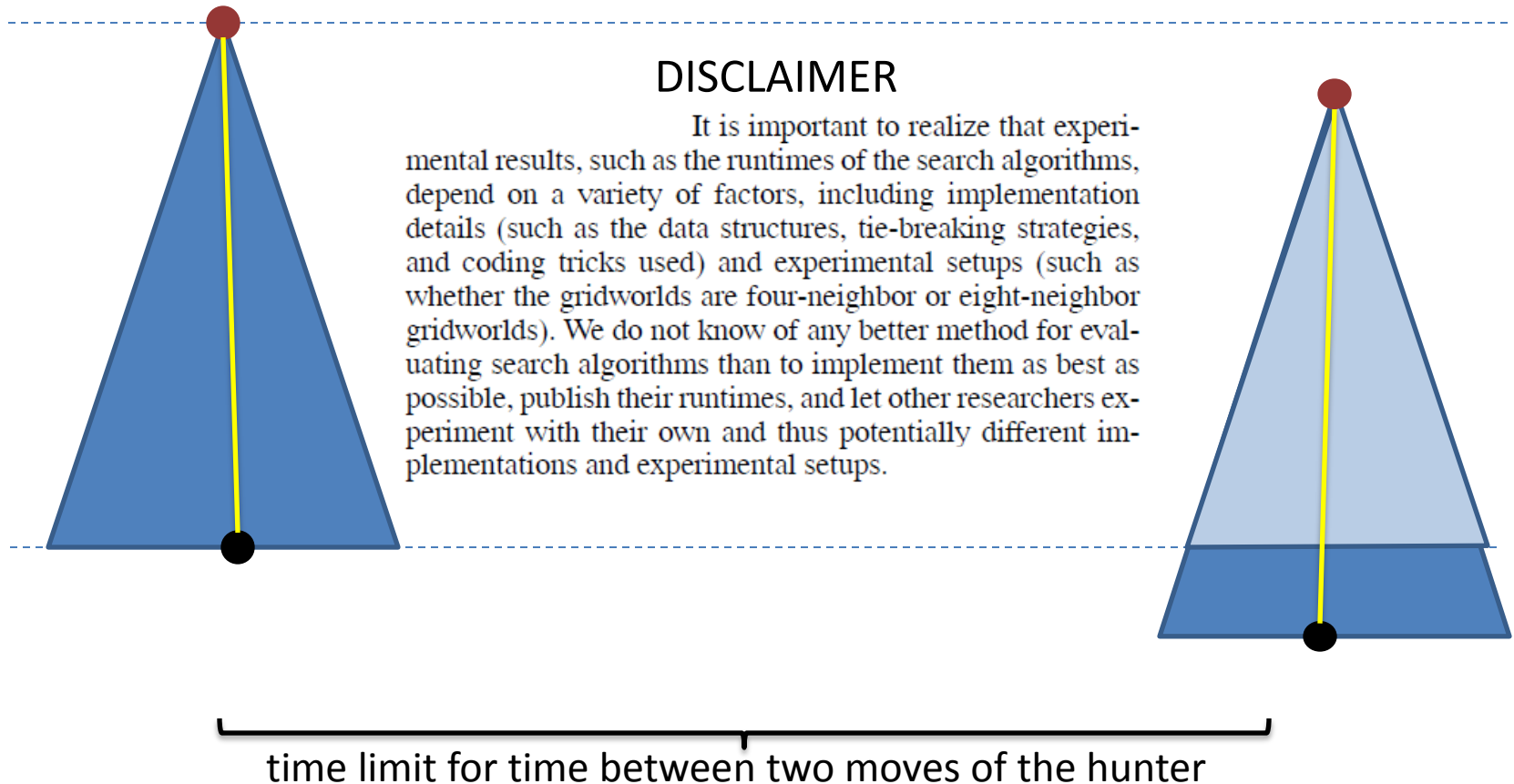


FRA*



FRA*

- The hunter uses incremental A*.



WA^* [Pohl, 1970]

- Idea 2:
 - Reduce the runtime of the A^* search with weighted A^* search

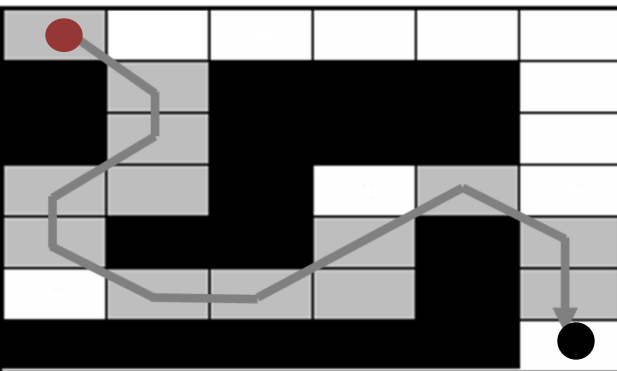
WA*

- The hunter uses weighted A*.

The smaller the weight w , the slower the search but the shorter the path.

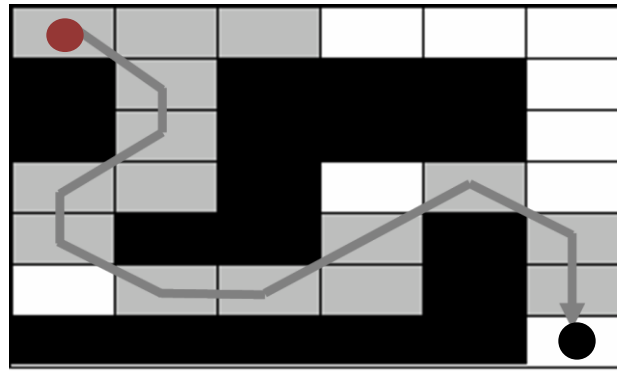
Weighted A* with weight one is identical to A*.

$$f(s) = g(s) + w h(s)$$



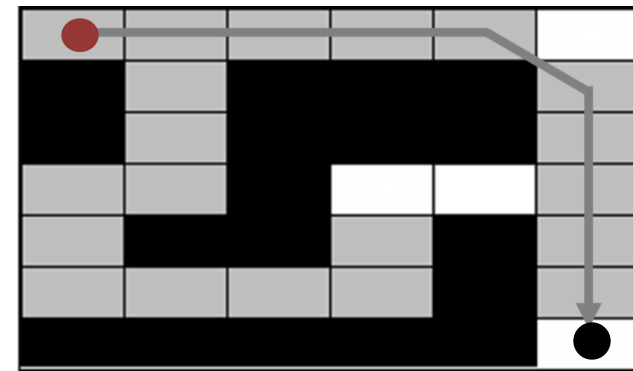
$w=2.5$

13 expansions
11 movements



$w=1.5$

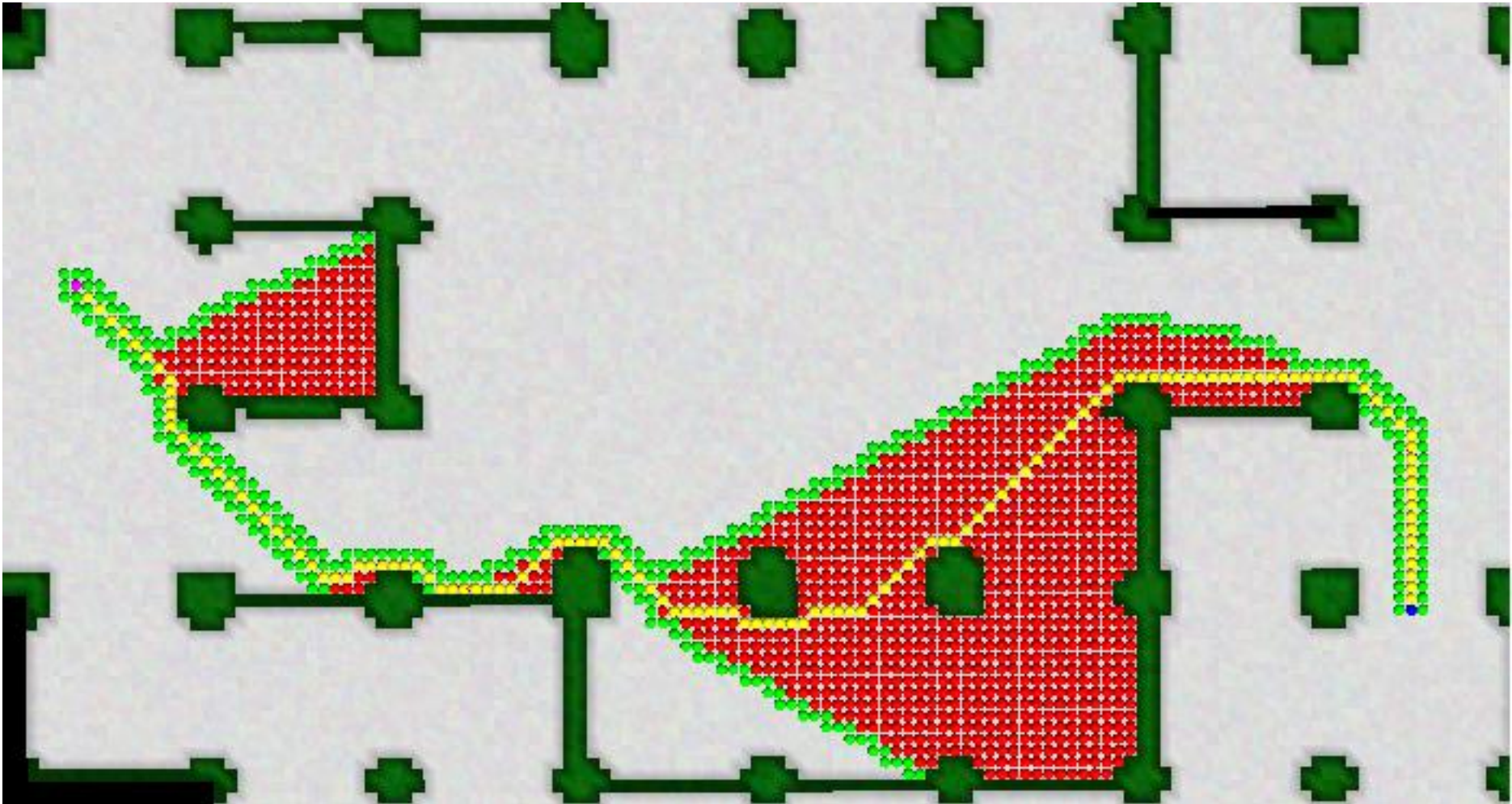
15 expansions
11 movements



$w=1.0$

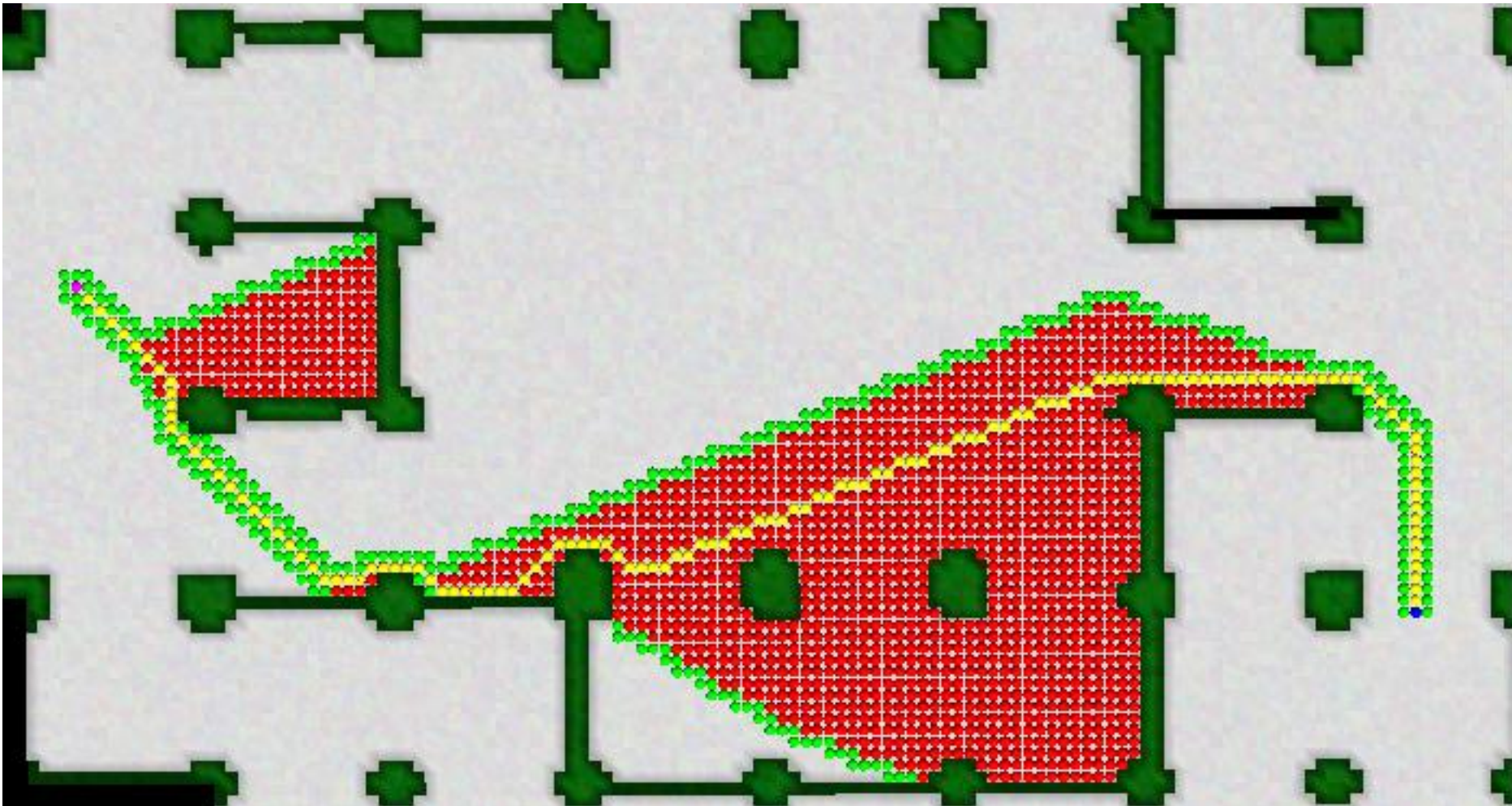
20 expansions
10 movements

WA^*



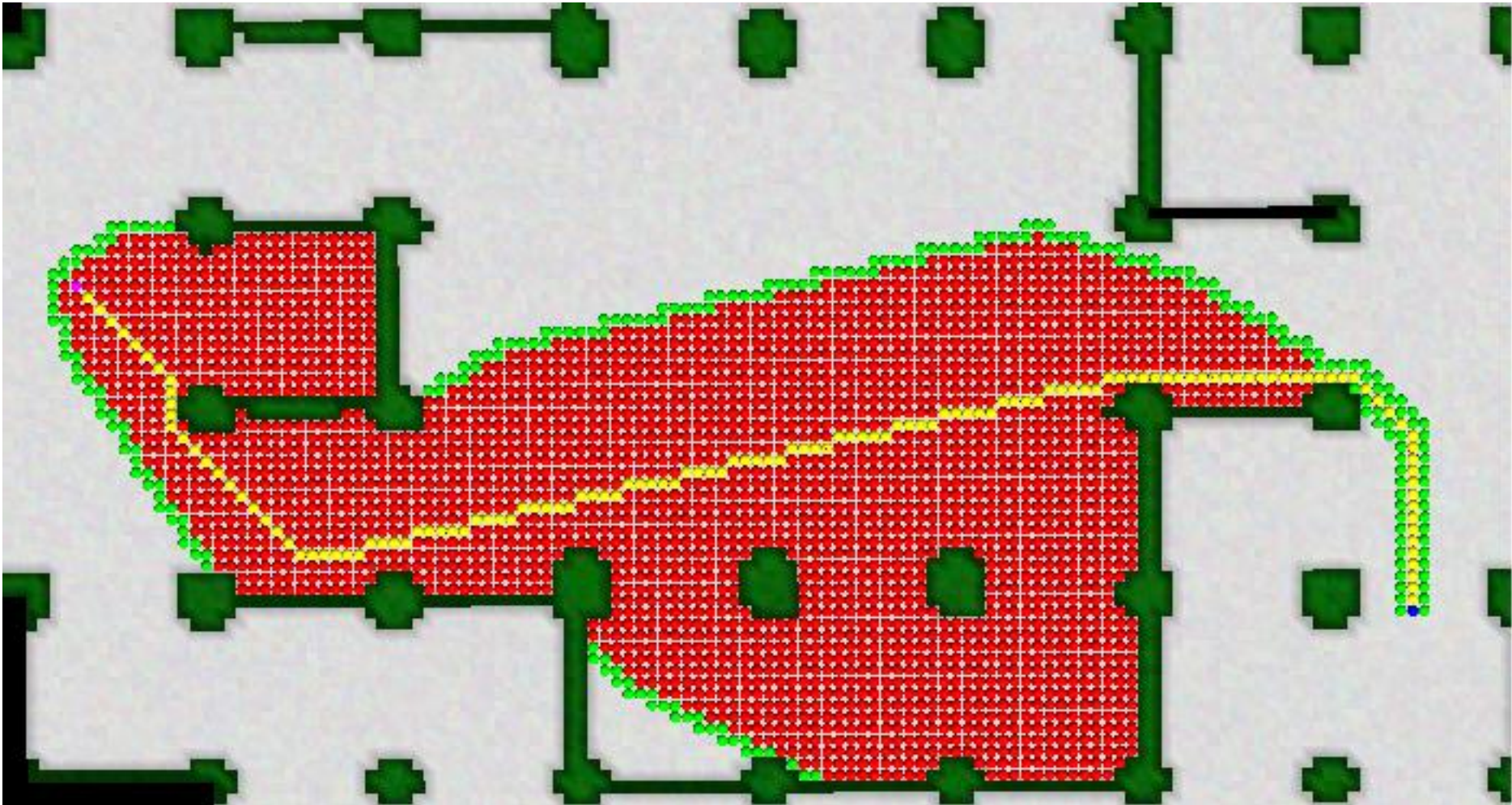
$w=1.6$

WA^*



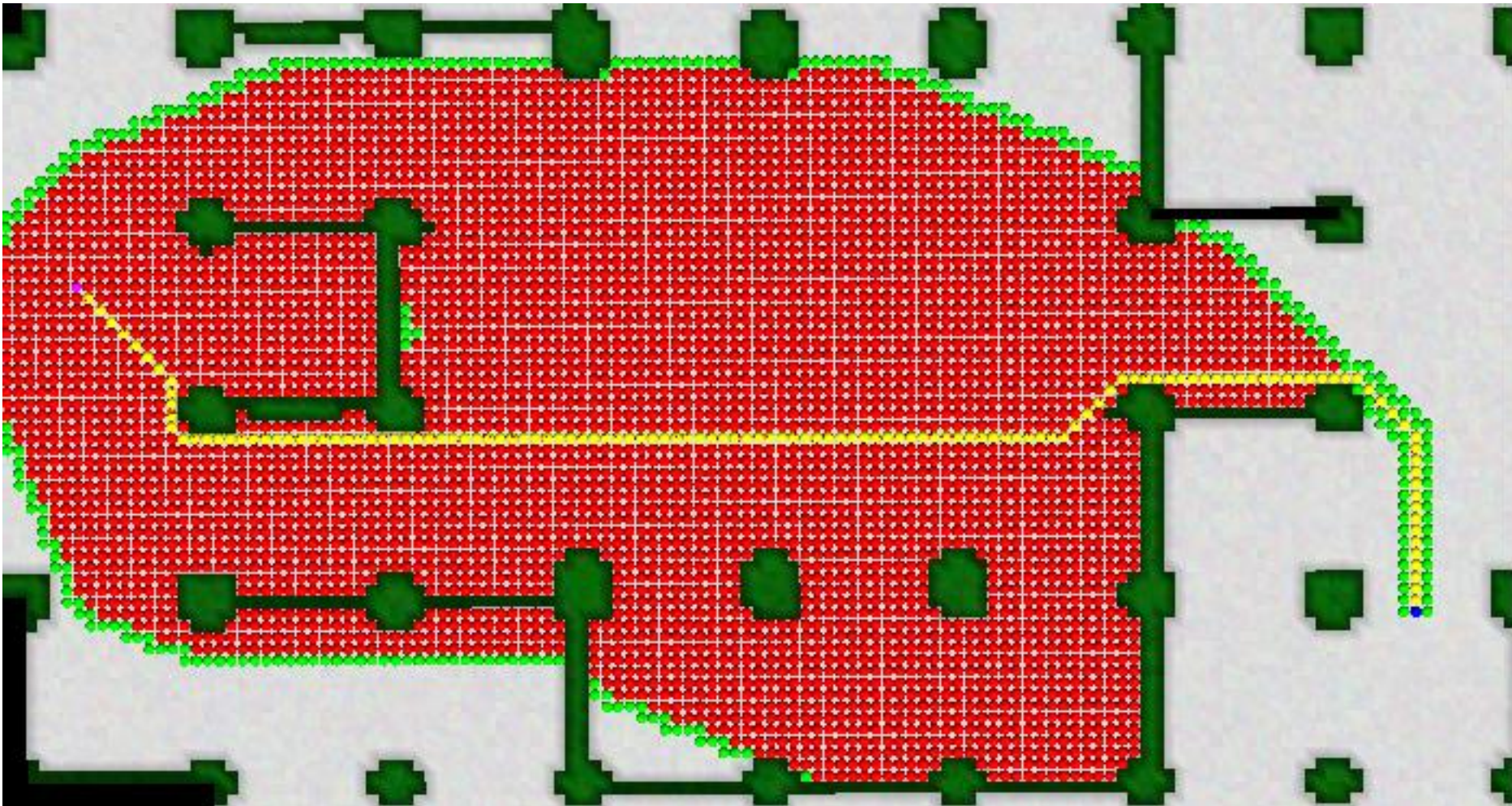
$w=1.4$

WA^*



$w=1.2$

WA^*



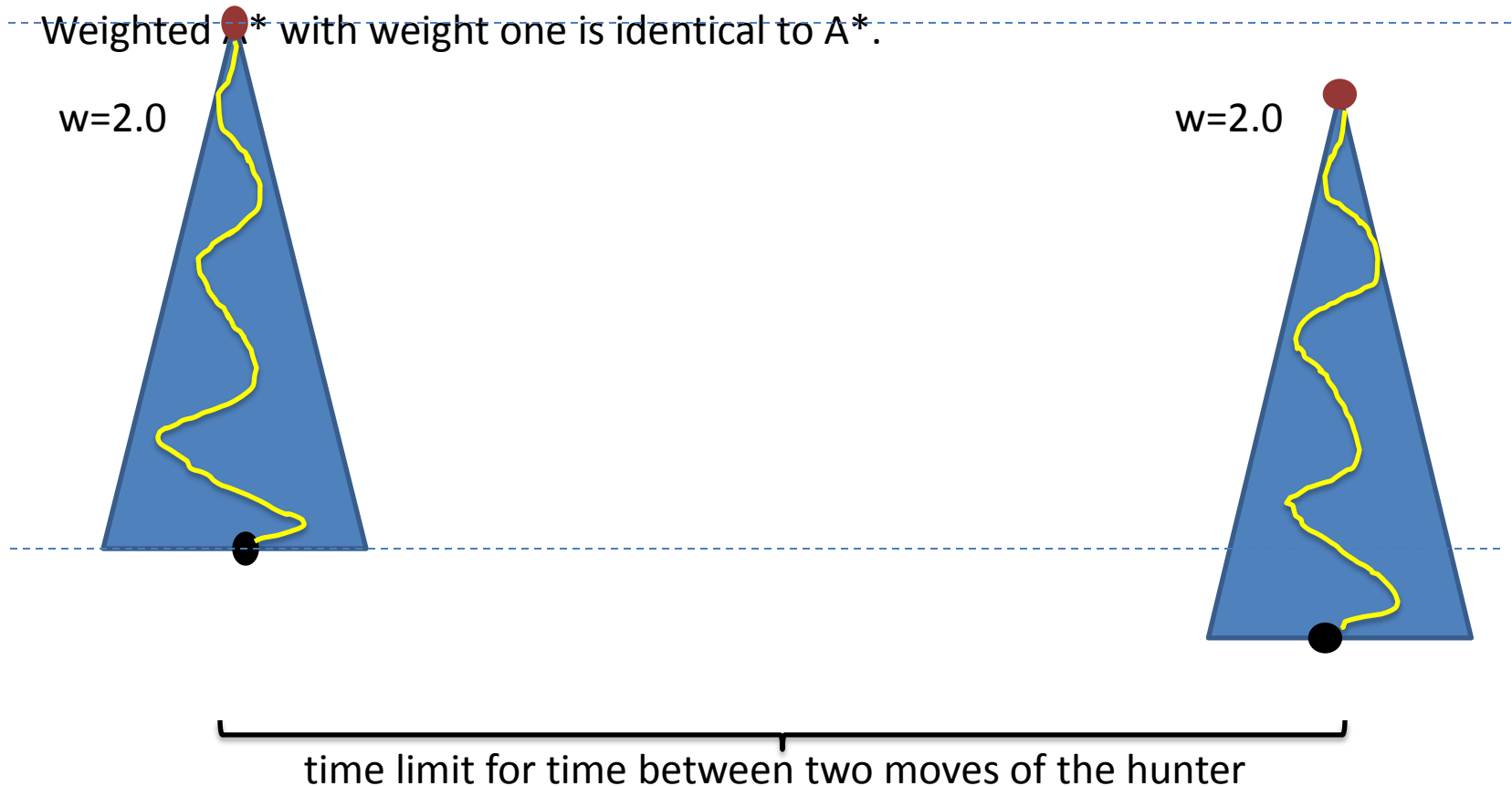
$w=1.0$

WA*

- The hunter uses weighted A*.

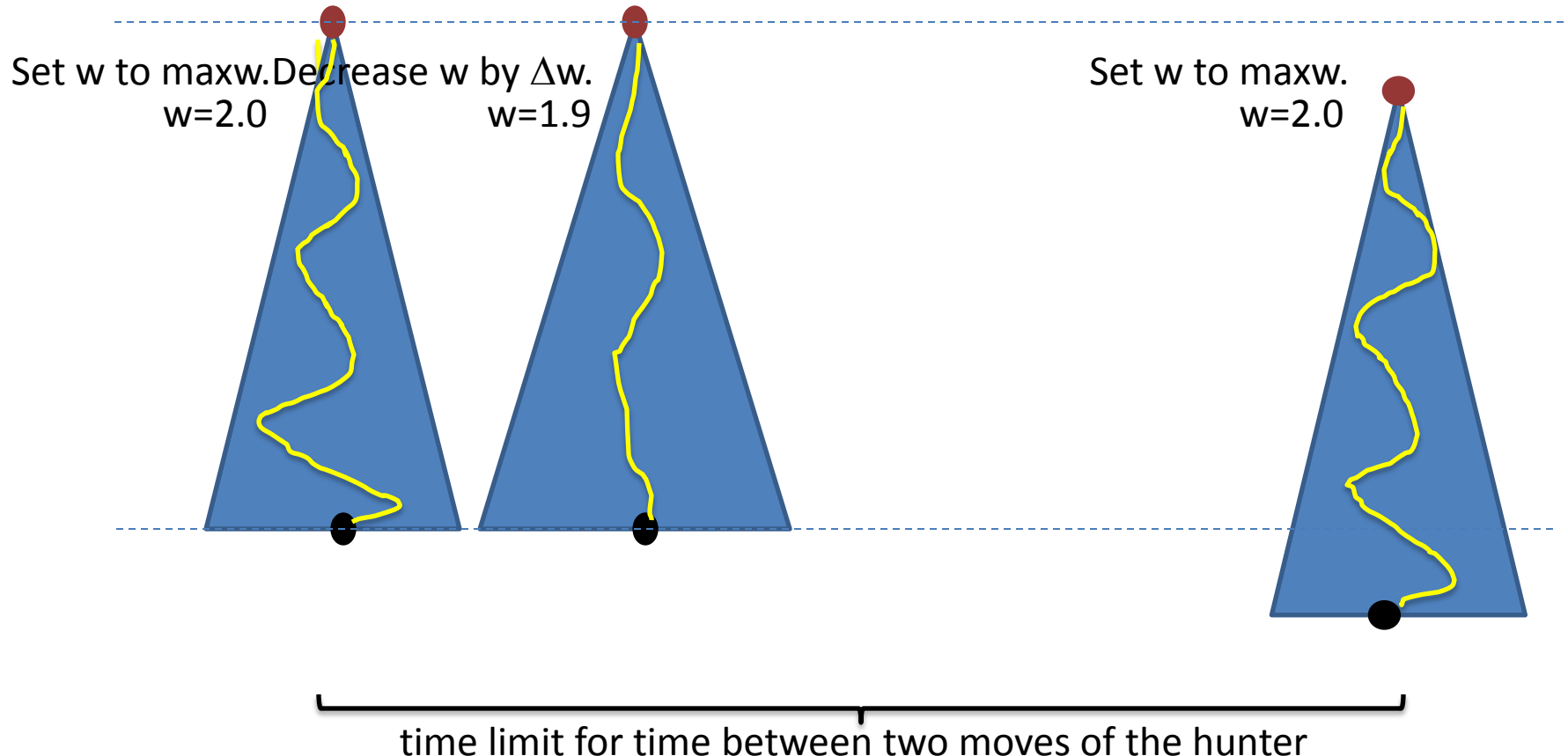
The smaller the weight w , the slower the search but the shorter the path.

Weighted A* with weight one is identical to A*.



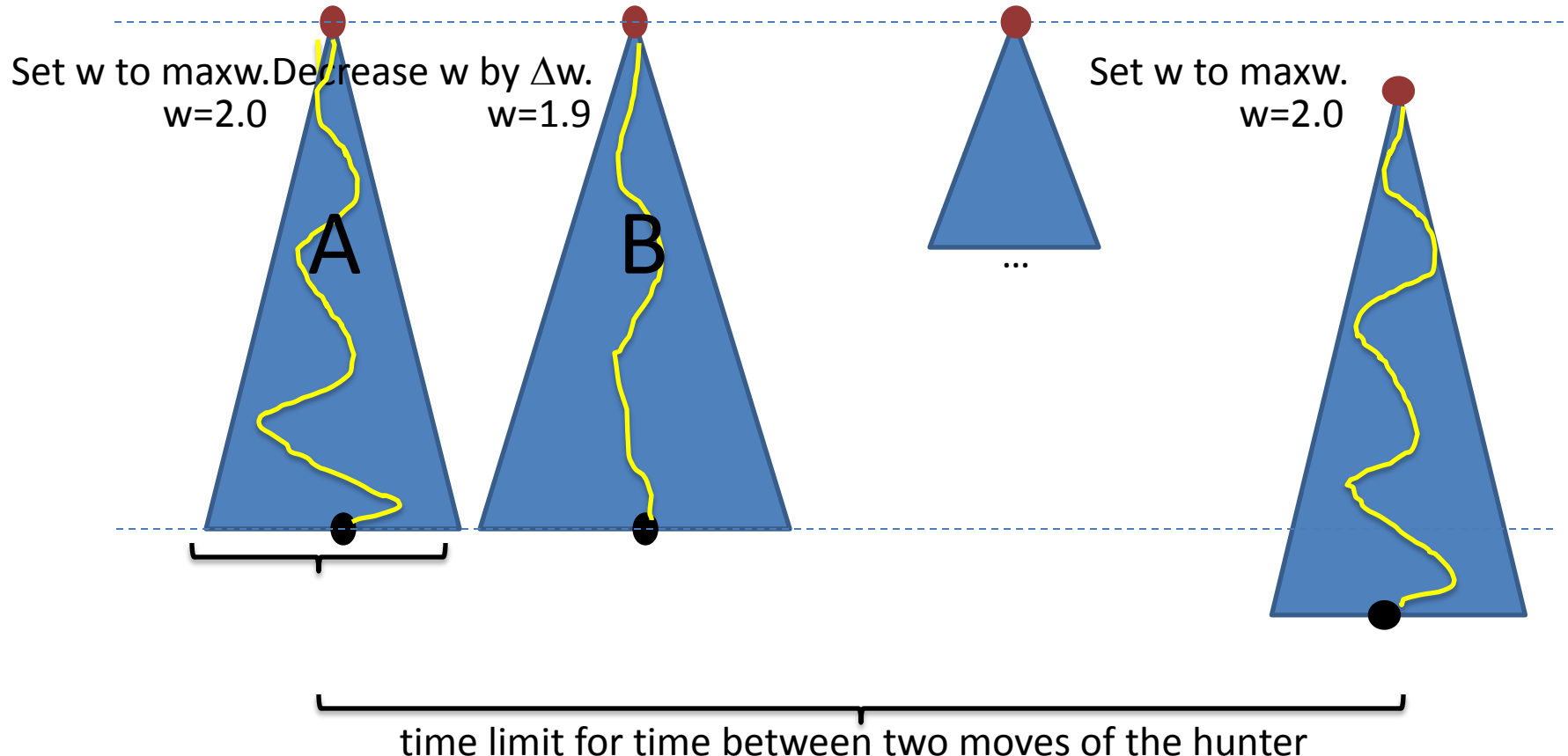
Repeated WA*

- The hunter uses weighted A* repeatedly, where the weight decreases over time until it is one.



Repeated WA*

- The hunter uses weighted A^* repeatedly, where the weight decreases over time until it is one.



ARA*

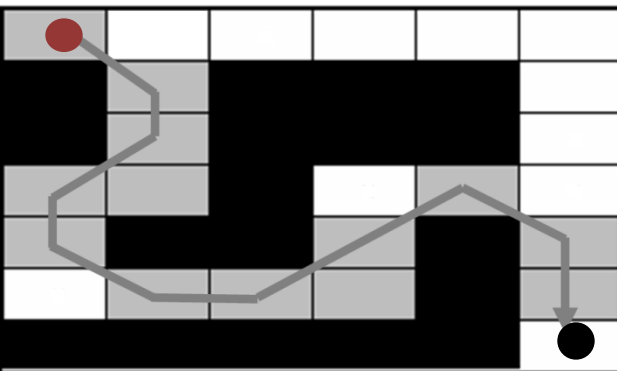
[Likhachev, Gordon, Thrun, 2003]

- The hunter uses weighted A*.

The smaller the weight w , the slower the search but the shorter the path.

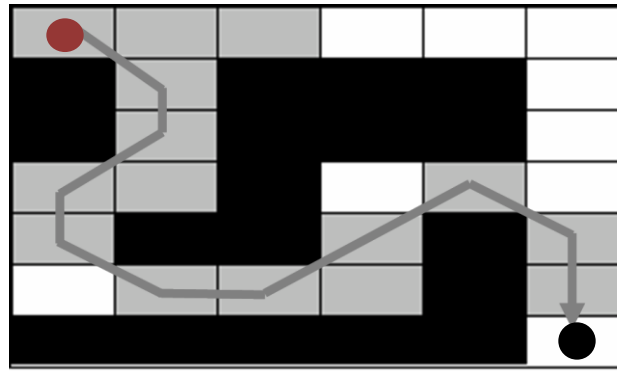
Weighted A* with weight one is identical to A*.

$$f(s) = g(s) + w h(s)$$



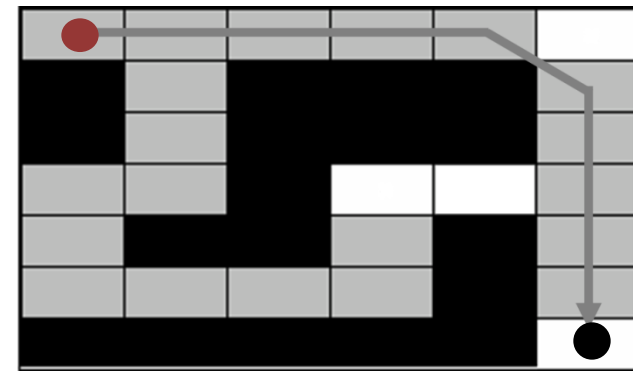
$w=2.5$

13 expansions
11 movements



$w=1.5$

15 expansions
11 movements



$w=1.0$

20 expansions
10 movements

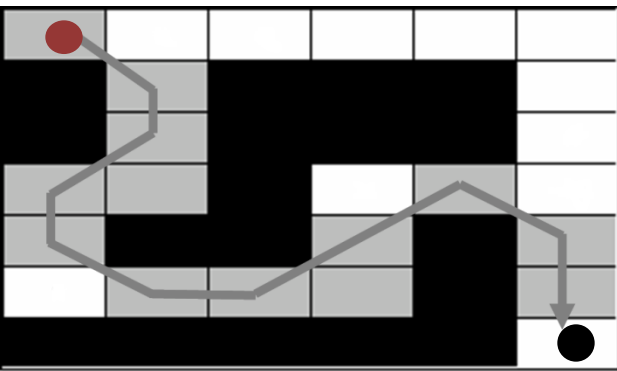
ARA*

- The hunter uses weighted A*.

The smaller the weight w , the slower the search but the shorter the path.

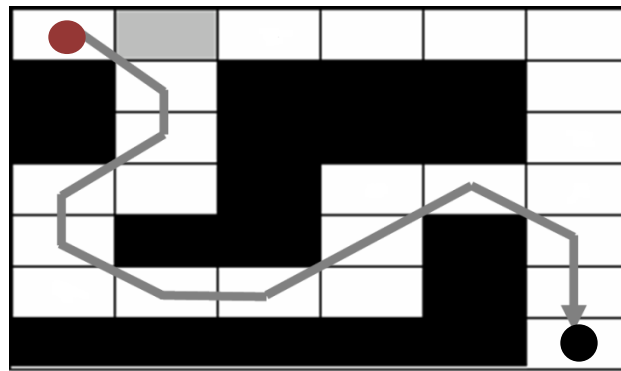
Weighted A* with weight one is identical to A*.

$$f(s) = g(s) + w h(s)$$



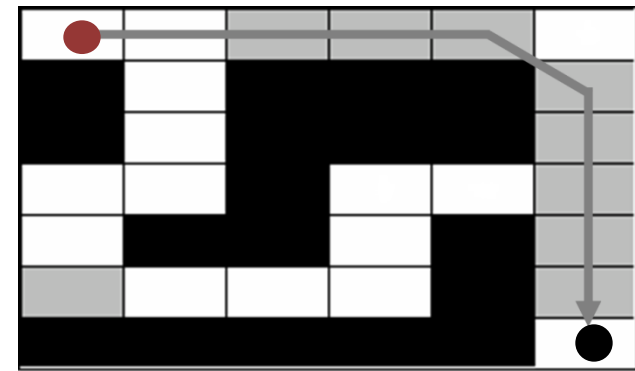
$w=2.5$

13 expansions
11 movements



$w=1.5$

1 expansion
11 movements



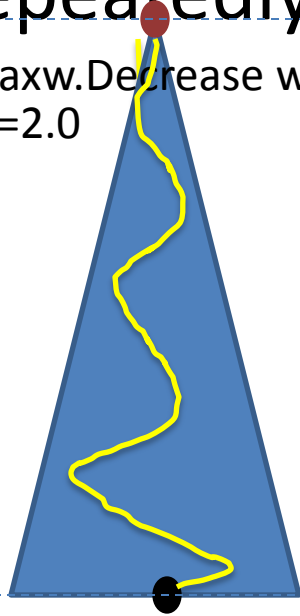
$w=1.0$

9 expansions
10 movements

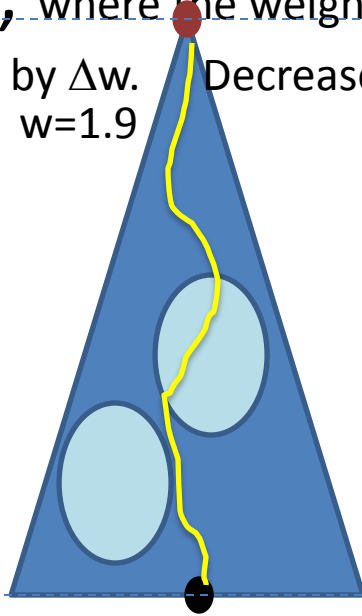
ARA*

- The hunter uses incremental weighted A^* repeatedly, where the weight decreases over time until it is one.

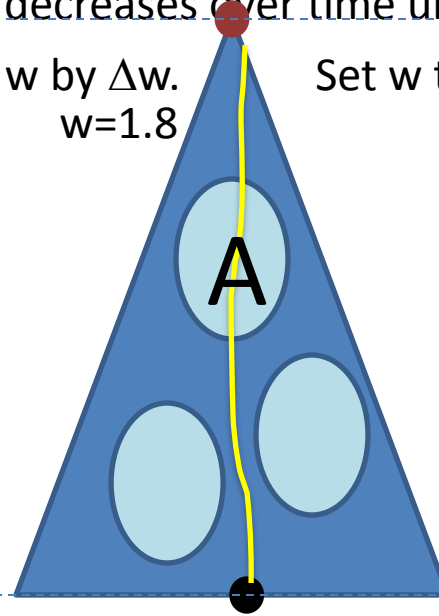
Set w to $\max w$.
 $w=2.0$



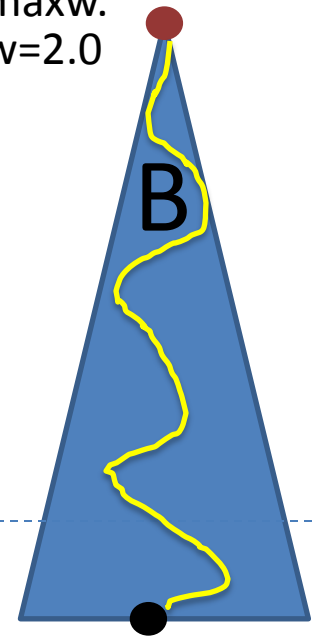
Decrease w by Δw .
 $w=1.9$



Decrease w by Δw .
 $w=1.8$

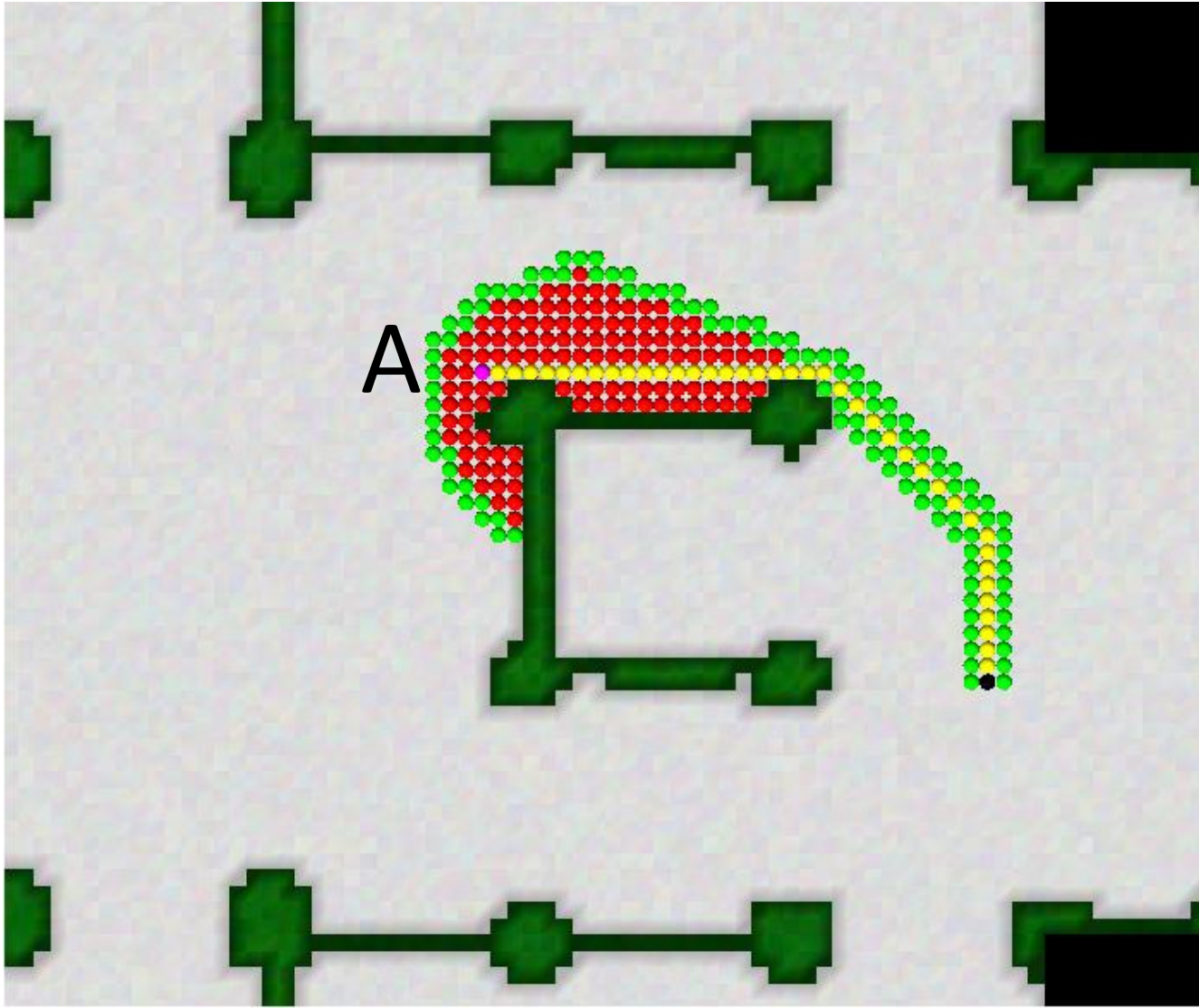


Set w to $\max w$.
 $w=2.0$

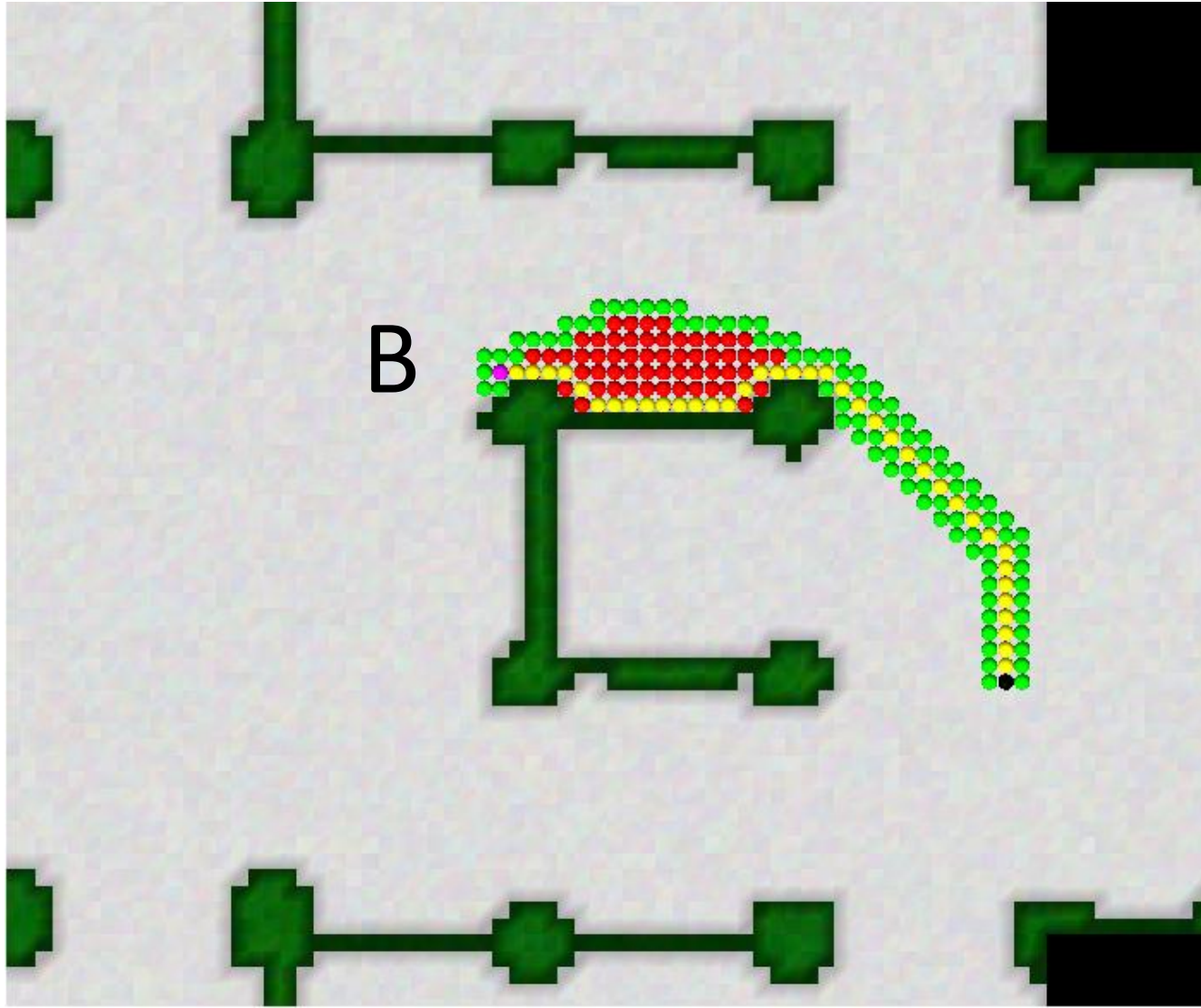


time limit for time between two moves of the hunter

$$\text{Incremental ARA}^* = \text{FRA}^* + \text{ARA}^*$$



Incremental $ARA^* = FRA^* + ARA^*$

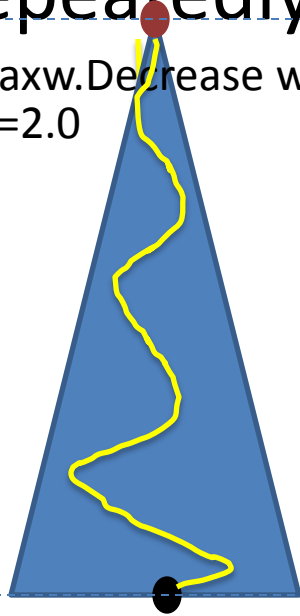


ARA*

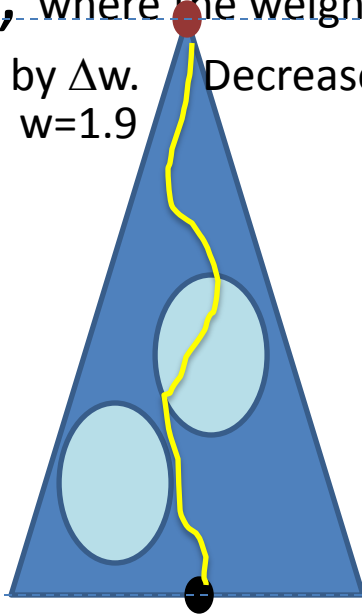
[Likhachev, Gordon, Thrun, 2003]

- The hunter uses incremental weighted A* repeatedly, where the weight decreases over time until it is one.

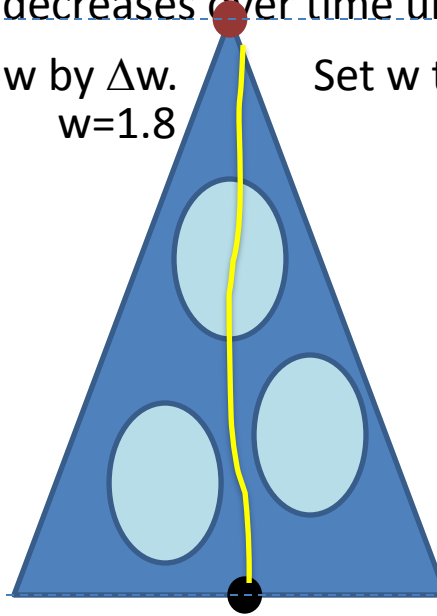
Set w to $\max w$.
 $w=2.0$



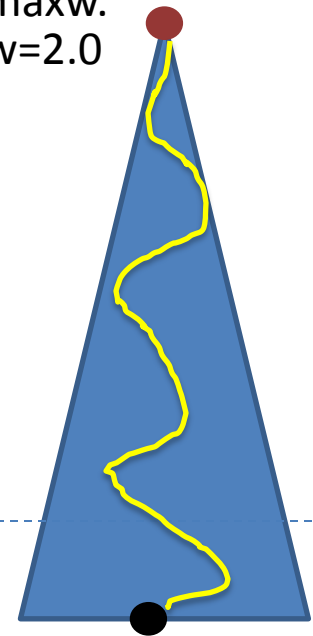
Decrease w by Δw .
 $w=1.9$



Decrease w by Δw .
 $w=1.8$



Set w to $\max w$.
 $w=2.0$



time limit for time between two moves of the hunter

$$\text{Incremental ARA}^* = \text{FRA}^* + \text{ARA}^*$$

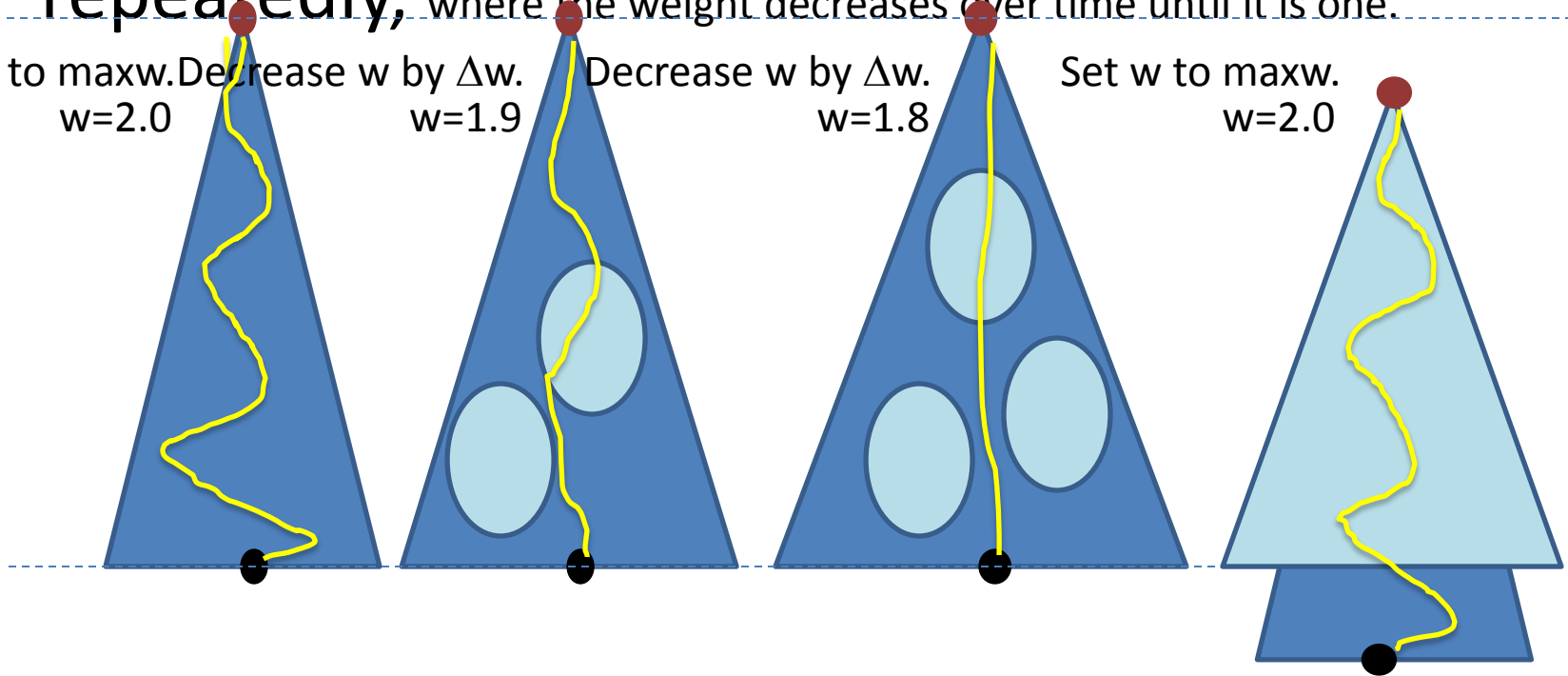
- The hunter uses incremental weighted A^* repeatedly, where the weight decreases over time until it is one.

Set w to max w .
Decrease w by Δw .
 $w=2.0$

Decrease w by Δw .
 $w=1.9$

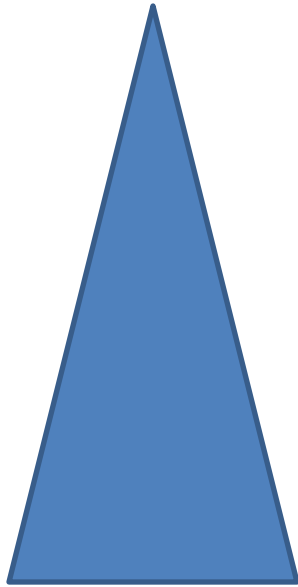
Decrease w by Δw .
 $w=1.8$

Set w to max w .
 $w=2.0$

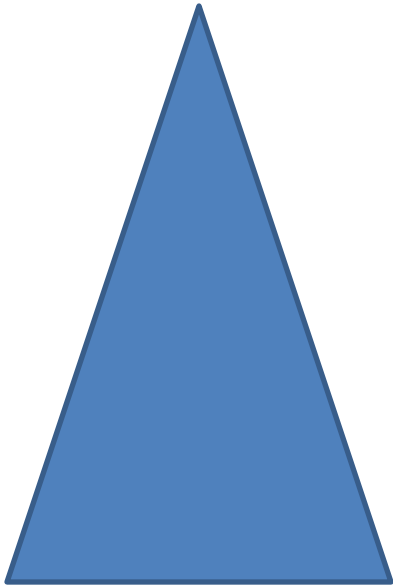


time limit for time between two moves of the hunter

$$\text{Incremental ARA}^* = \text{FRA}^* + \text{ARA}^*$$



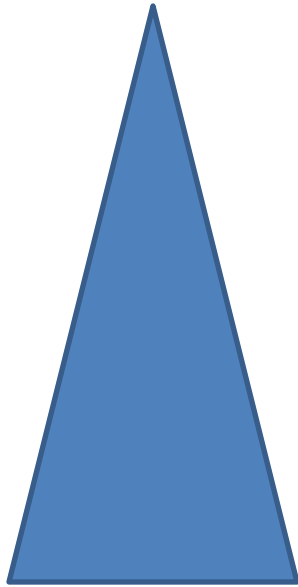
FRA*



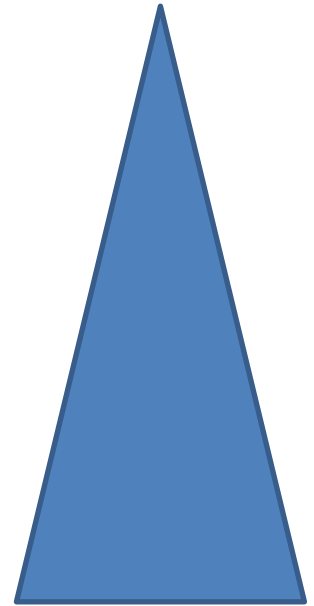
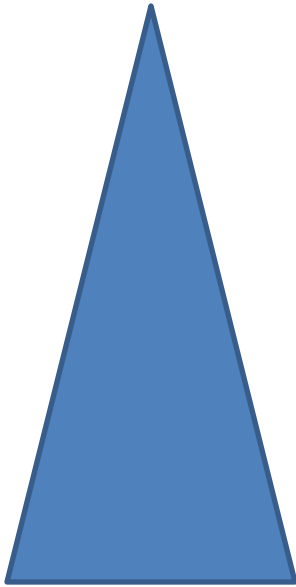
^



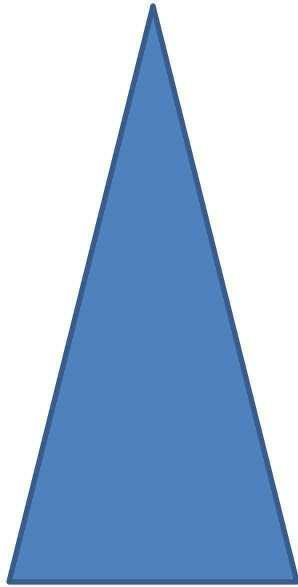
$$\text{Incremental ARA}^* = \text{FRA}^* + \text{ARA}^*$$



ARA*



$$\text{Incremental ARA}^* = \text{FRA}^* + \text{ARA}^*$$



Incremental ARA* = FRA* + ARA*

```

01 function ImprovePath()
02 while  $g(s_{goal}) + \epsilon \times h(s_{goal}, s_{goal}) > \min_{s \in OPEN} (g(s) + \epsilon \times h(s, s_{goal}))$ 
03   move  $s \in OPEN$  with the smallest  $g(s) + \epsilon \times h(s, s_{goal})$  from  $OPEN$  to  $CLOSED$ ;
04    $v(s) := g(s)$ ;
05   forall  $s' \in Neighbor(s)$ 
06     if  $g(s') > v(s) + c(s, s')$ 
07        $g(s') := v(s) + c(s, s')$ ;
08        $parent(s') := s$ ;
09       if  $s' \notin CLOSED$ 
10         if  $s' \notin OPEN$  AND  $s' \notin INCONS$ 
11           insert  $s'$  into  $OPEN$ ;
12     else
13       move  $s'$  from  $CLOSED$  to  $INCONS$ ;
14 if  $g(s_{goal}) = \infty$ 
15   return false;
16 else
17   return true;
18 function ComputePath()
19 repeat
20   return false if ImprovePath() = false;
21   return true if  $\epsilon = 1$  OR the time limit has been reached;
22    $OPEN := OPEN \cup INCONS$ ;
23    $CLOSED := INCONS := \emptyset$ ;
24    $\epsilon := \max(1, \epsilon - \delta_\epsilon)$ ;
25 procedure Step1()
26 if  $g(s_{start}) \neq v(s_{start})$ 
27    $g(s_{start}) := v(s_{start})$ ;
28   delete  $s_{start}$  from  $INCONS$  if  $s_{start} \in INCONS$ ;
29   delete  $s_{start}$  from  $OPEN$  if  $s_{start} \in OPEN$ ;
30 procedure Step2()
31 if  $s_{start} \neq previous\_s_{start}$ 
32    $parent(s_{start}) := NULL$ ;
33   forall  $s$  in the search tree rooted at  $previous\_s_{start}$  but not in the subtree rooted at  $s_{start}$ 
34      $v(s) := g(s) := \infty$ ;
35      $parent(s) := NULL$ ;
36   delete  $s$  from  $INCONS$  if  $s \in INCONS$ ;
37   delete  $s$  from  $OPEN$  if  $s \in OPEN$ ;
38   insert  $s$  into  $DELETED$ ;

```

```

39 procedure Step3()
40 forall  $s \in DELETED$ 
41   forall  $s' \in Neighbor(s)$ 
42     if  $g(s) > v(s') + c(s', s)$ 
43        $g(s) := v(s') + c(s', s)$ ;
44        $parent(s) := s'$ ;
45   if  $g(s) \neq \infty$ 
46     insert  $s$  into  $OPEN$ ;
47  $OPEN := OPEN \cup INCONS$ ;
48  $CLOSED := INCONS := DELETED := \emptyset$ ;
49 procedure Step4()
50 if  $g(s_{goal}) + \epsilon \times h(s_{goal}, s_{goal}) > \min_{s \in OPEN} (g(s) + \epsilon \times h(s, s_{goal}))$ 
51    $\epsilon := \epsilon_{max}$ ;
52 else
53    $\epsilon := \max(1, \epsilon - \delta_\epsilon)$ ;
54 function Main()
55 forall  $s \in S$ 
56    $v(s) := g(s) := \infty$ ;
57    $parent(s) := NULL$ ;
58  $\epsilon := \epsilon_{max}$ ;
59  $s_{start} :=$  the current cell of the hunter;
60  $s_{goal} :=$  the current cell of the target;
61  $OPEN := CLOSED := INCONS := DELETED := \emptyset$ ;
62  $g(s_{start}) := 0$ ;
63 insert  $s_{start}$  into  $OPEN$ ;
64 while  $s_{start} \neq s_{goal}$ 
65   return false if ComputePath() = false; /* Step 5 */
66    $previous\_s_{start} := s_{start}$ ;
67   identify a path from  $s_{start}$  to  $s_{goal}$  using the parents;
68   while the target has not been caught yet AND is still on the path from  $s_{start}$  to  $s_{goal}$ 
69     the hunter follows the path from  $s_{start}$  to  $s_{goal}$ ;
70   return true if the target has been caught;
71    $s_{start} :=$  the current cell of the hunter;
72    $s_{goal} :=$  the current cell of the target;
73   Step1(); /* Step 1 */
74   Step2(); /* Step 2 */
75   Step3(); /* Step 3 */
76   Step4(); /* Step 4 */
77 return true;

```

Incremental $ARA^* = FRA^* + ARA^*$

- The algorithm:
 - Make the new state of the hunter locally consistent.
 - Delete all states from the search tree that are not in the subtree rooted in the new state of the hunter.
 - Add to the OPEN list all states that border non-leaf states in the search tree.
 - If the f-value of the new state of the target is no larger than the smallest f-values of all states in the OPEN list (= the search tree already contains a w-suboptimal path from the state of the hunter to the state of the target), then decrease w to $\max(1, w - \Delta w)$. Otherwise, set w to maxw.
 - Run an ARA^* search to find an w-suboptimal path from the state of the hunter to the state of the target.
 - Move the hunter along the path until it catches the target or the target moves off the path. In the former case, stop. In the latter case, repeat.

Experimental Results

- We use 100 test cases with randomly selected unblocked connected cells for the hunter and target, namely
 - 100 four-neighbor random maps of size 1,000x1,000 with 25 percent randomly blocked cells; and
 - one four-neighbor game map of size 626x626 adapted from Warcraft III
- The target repeatedly follows a shortest path from its current cell to a randomly selected unblocked cell, skipping every tenth move.

Experimental Results

- FRA*, Repeated ARA* and Incremental ARA* were implemented in similar ways (e.g. using a binary heap).
- There is a time limit for each search but we allow the search algorithms to exceed the time limit until they find some path from the hunter to the target.
- For Repeated ARA* and Incremental ARA*, we use $\text{maxw}=2.0$ and $\Delta w=0.1$.

Experimental Results (Random Maps)

	Time limit	Moves per test case	First searches exceeding the time limit	Searches per time interval
FRA*	200μs	747	17.6%	-
Repeated ARA*	200μs	868	65.2%	3.29
Incremental ARA*	200μs	827	6.8%	5.47
FRA*	500μs	747	12.8%	-
Repeated ARA*	500μs	852	28.4%	5.53
Incremental ARA*	500μs	804	0.5%	7.55
FRA*	1000μs	747	3.6%	-
Repeated ARA*	1000μs	836	4.1%	7.82
Incremental ARA*	1000μs	785	0.2%	9.07

Experimental Results (Game Map)

	Time limit	Moves per test case	First searches exceeding the time limit	Searches per time interval
FRA*	200μs	545	15.5%	-
Repeated ARA*	200μs	652	69.1%	3.29
Incremental ARA*	200μs	613	6.1%	5.47
FRA*	500μs	545	10.0%	-
Repeated ARA*	500μs	645	49.0%	4.69
Incremental ARA*	500μs	596	0.7%	7.51
FRA*	1000μs	545	5.7%	-
Repeated ARA*	1000μs	639	34.3%	5.97
Incremental ARA*	1000μs	577	0.3%	8.87

Conclusions

- Replanning is important when the search problem changes.
- We begin to understand replanning for goal-directed navigation in **unknown** terrain towards a **stationary** target (where the hunter gains knowledge about the terrain).
- This paper studies replanning for goal-directed navigation in **known** terrain towards a **moving** target (where the hunter gains knowledge about the target cell).

Conclusions

- Incremental ARA* is the first incremental anytime search algorithm for moving target search in known terrain.
- Incremental ARA* can be used with smaller time limits between moves of the hunter than competing state-of-the-art moving target search algorithms.

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