

# Optimally Relaxing Partial-Order Plans with MaxSAT

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June 27, 2012

- Partial-Order Plans (POPs) have an appealing *least commitment* nature.
- POP planners are not as effective as sequential ones.
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## Goal

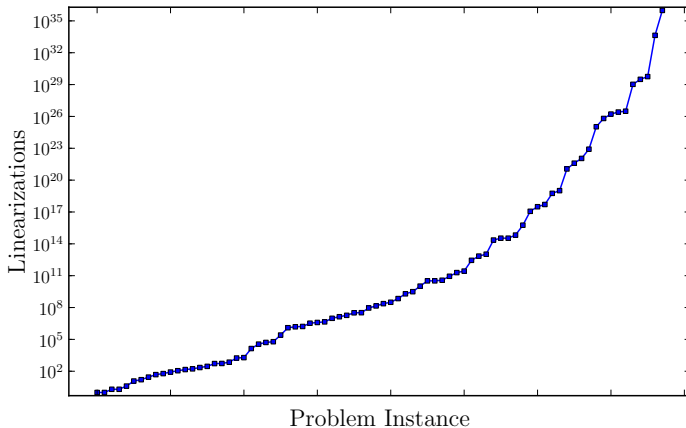
Can we use a sequential planner to generate a plan and then use a SAT solver to turn that plan into an “optimal” POP?

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- 2 Encode the problem of finding a POP from the plan.
- 3 Use a MaxSAT solver to compute a POP (Sat4j).

# The Result



- 1 Background
- 2 Least Commitment Criteria
- 3 Encoding
- 4 Empirical Evaluation
- 5 Conclusion



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- 1 Background
  - Propositional Planning
  - Partial Order Plans
  - Partial Weighted MAXSAT

## Planning Problem

- STRIPS Planning problem  $\Pi = \langle F, O, I, G \rangle$
- $F$ : Finite set of fluents
- $O$ : Finite set of operators
- $I$ : Initial state ( $I \subseteq F$ )
- $G$ : Goal state ( $G \subseteq F$ )

# Propositional Planning

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## State

A state  $s \subseteq F$  is a subset of the fluents that currently hold. In a *complete state*, fluents not in  $s$  are presumed to be false. A *partial state* does not have this assumption.

For each  $o \in O$

- $PRE(o) \subseteq F$ : Precondition
- $ADD(o) \subseteq F$ : Add effects
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## Action

An instance of an operator is referred to as an *action*. There may be many actions that correspond to the same operator.

## Action execution

- An action  $a$  is *executable* in state  $s$  iff  $PRE(a) \subset s$ .
- Executing an action  $a$  executable in state  $s$  causes the state to change to  $(s \setminus DEL(a)) \cup ADD(a)$ .
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## Sequential Plan

A *sequential plan* is a sequence of actions  $\vec{a} = [a_1, a_2, \dots, a_n]$  that can be executed in the initial state  $I$ , and achieves the goal  $G$ .

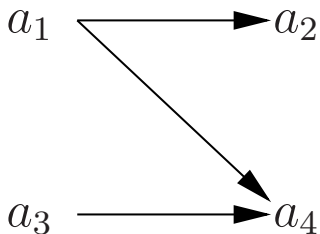


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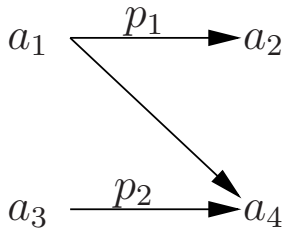
# POP Example

$a_1$  $a_2$  $a_3$  $a_4$ 

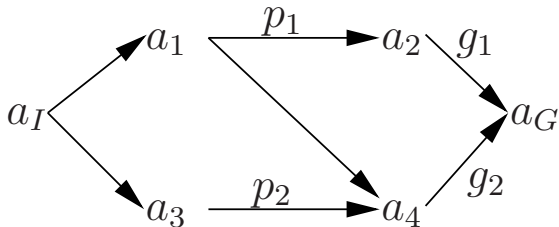
Actions



Ordering Constraints



Causal Links



Actions for  $A$  and  $G$

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- $\mathcal{C}$ : Set of causal links between the actions in  $\mathcal{A}$ . A causal link is an annotated ordering constraint that is labelled with a fluent that represents why the link exists.
  - E.g.,  $(a_1 \overset{f}{\prec} a_2) \in \mathcal{C}$  (can assume  $f \in ADD(a_1) \cap PRE(a_2)$ )

## Linearizations

A *linearization* of the POP  $P = \langle \mathcal{A}, \mathcal{O}, \mathcal{C} \rangle$  is a total ordering of actions in  $\mathcal{A}$  that respects the ordering constraints of  $\mathcal{O}$ .

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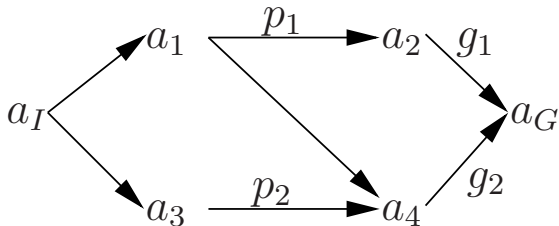
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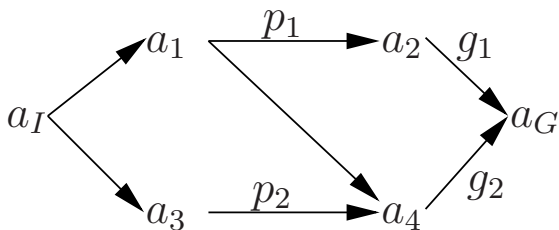
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- A causal link  $(a_1 \overset{f}{\prec} a_2)$  is *threatened* if there is some action  $a_3$  such that  $f \in DEL(a_3)$  and  $\mathcal{O} \cap \{(a_3 \prec a_1), (a_2 \prec a_3)\} = \emptyset$ .

# POP Example

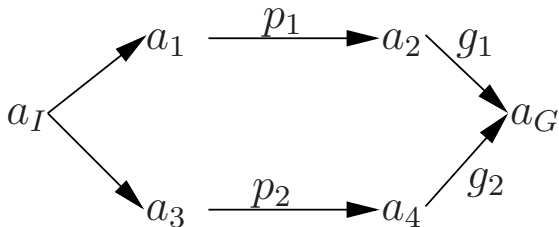




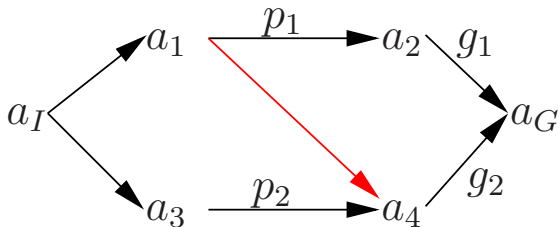
- Linearizations:

$[a_1, a_2, a_3, a_4]$	$[a_1, a_3, a_2, a_4]$	$[a_1, a_3, a_4, a_2]$
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## Threat & Support Validity

A POP  $P = \langle \mathcal{A}, \mathcal{O}, \mathcal{C} \rangle$  is valid for  $\Pi$  iff the following holds:

- 1 There are no open preconditions in  $P$ .
- 2 No causal link in  $\mathcal{C}$  is threatened.
- 3  $\mathcal{A}$  contains the dummy actions  $a_I$  and  $a_G$ .

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## Satisfiability (SAT)

$$(x \vee y) \wedge (\neg x \vee z)$$

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$$\begin{aligned} & (x \vee y \vee \neg z) \wedge (x \vee z) \wedge (y \vee z) \wedge \\ & (\neg x \vee \neg y) \wedge (\neg z \vee \neg x) \wedge (\neg z \vee \neg y) \end{aligned}$$

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## Weighted MAXSAT

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## Partial Weighted MAXSAT

$$\overset{1}{(x)} \wedge \overset{2}{(y)} \wedge \overset{3}{(z)} \wedge (\neg x \overset{\bullet}{\vee} \neg z) \wedge (\neg y \overset{\bullet}{\vee} \neg z) \wedge (\neg x \overset{\bullet}{\vee} \neg y)$$

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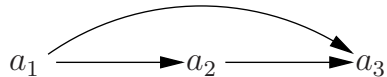
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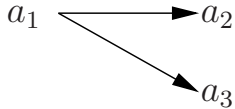
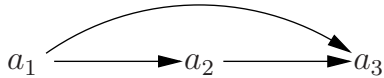


- 2 Least Commitment Criteria
  - Deordering & Reordering
  - Least Commitment POP

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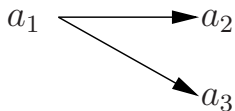


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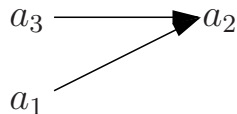


Deordering

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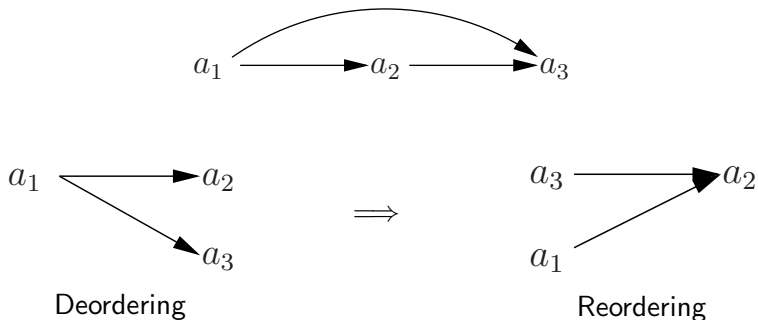


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# Least Commitment POP

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## Least Commitment POP (LCP)

Let  $P = \langle \mathcal{A}, \mathcal{O} \rangle$  and  $Q = \langle \mathcal{A}', \mathcal{O}' \rangle$  be two POPs valid for  $\Pi$ .  $Q$  is a *least commitment POP* (LCP) of  $P$  iff  $Q$  is the minimum reordering of itself and there is no valid POP  $\langle \mathcal{A}'', \mathcal{O}'' \rangle$  for  $\Pi$  such that  $\mathcal{A}'' \subseteq \mathcal{A}$  and  $|\mathcal{A}''| < |\mathcal{A}'|$ .

- 1 Background
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- 3 Encoding
  - Core Encoding
  - Extensions
  - Approach

## Action Variables and Ordering Variables

- $\forall a \in \vec{a}, \quad x_a$  : True iff  $a$  is in the final POP.
- $\forall a_i, a_j \in \vec{a}, \quad \kappa(a_i, a_j)$ : True iff  $(a_i \prec a_j)$  is in the final POP.
- $\Upsilon(a_i, a_j, p)$ :  $a_i$  supports  $a_j$  with  $p$ .

## Basic Clauses

- No self loops.
- Include  $a_I$  and  $a_G$ .
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- $\kappa(a_i, a_j) \wedge \kappa(a_j, a_k) \rightarrow \kappa(a_i, a_k)$  Transitive closure.

## POP Viability Clauses

- Ensure that if we include action  $a_j$ , then every precondition  $p$  of  $a_j$  must be satisfied by at least one achiever  $a_i$ .
- Ensure that if  $a_i$  achieves precondition  $p$  for action  $a_j$ , then no deleter of  $p$  will be allowed to occur between  $a_i$  and  $a_j$ .



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## Soft Clauses

- $w(\neg\kappa(a_i, a_j)) = 1, \quad \forall a_i, a_j \in \mathcal{A}$
- $w(\neg x_a) = 1 + |\mathcal{A}|^2, \quad \forall a \in \mathcal{A} \setminus \{a_I, a_G\}$

- 3 Encoding
  - Core Encoding
  - Extensions
  - Approach

## All Actions (AA)

$$(x_a), \quad \forall a \in \mathcal{A}$$

## Deordering (DO)

$$(\neg \kappa(a_j, a_i)), \quad [a_1, \dots, a_i, \dots, a_j, \dots, a_n]$$

## Variants

- AA, DO: Minimum Deordering (MD)
- AA,  $\neg$ DO: Minimum Reordering (MR)
- $\neg$ AA,  $\neg$ DO: Least Commitment POP (LCP)

- 3 Encoding
  - Core Encoding
  - Extensions
  - Approach

- 1 Generate a sequential plan (FF).
- 2 Encode the problem of finding a POP (MD, MR, or LCP).
- 3 Use a MAXSAT solver to compute the POP (Sat4j).

- 1 Background
- 2 Least Commitment Criteria
- 3 Encoding
- 4 Empirical Evaluation**
- 5 Conclusion



- 4 Empirical Evaluation
  - Relaxer Algorithm
  - Encoding Difficulty
  - POP Quality
  - Reordering Flexibility

# Relaxer Algorithm (KK)

Introduced by Kambhampati and Kedar (1994), the algorithm computes a deordering of a plan by removing redundant edges.

---

## Algorithm 1: Relaxer Algorithm

---

**Input:** Sequential plan,  $\vec{a}$ , including  $a_I$  and  $a_G$

**Output:** Partial-order plan,  $\langle \mathcal{A}, \mathcal{O}, \mathcal{C} \rangle$

```
1 foreach  $a_j \in \mathcal{A}$  do
2   foreach  $f \in PRE(a_j)$  do
3     Let  $a_i$  be the first action in  $\vec{a}$  such that  $i < j$ ,  $f \in ADD(a_i)$ , and
        $\forall k, i < k < j \Rightarrow f \notin DEL(a_k)$ .
4     Create a causal link between  $a_i$  and  $a_j$ .
5     Add necessary ordering constraints so  $f$  isn't threatened.
```

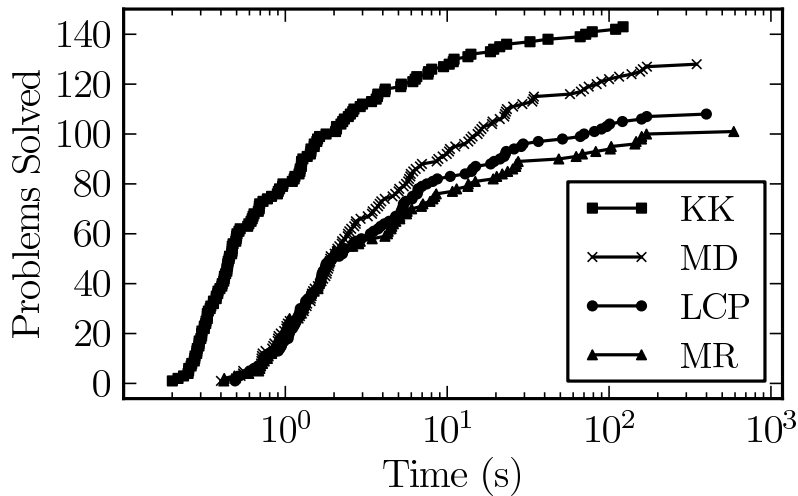
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- 4 Empirical Evaluation
  - Relaxer Algorithm
  - Encoding Difficulty
  - POP Quality
  - Reordering Flexibility

# Successfully Encoded

Domain	Num Probs	FF Solved	Successfully Encoded
Depots	22	22	22
Driverlog	20	16	16
Logistics	35	35	33
TPP	30	30	20
Rovers	20	20	20
Zeno	20	20	18
ALL	147	143	129

- Time / memory limit of 30min / 2GB.
- Encoding failure due to CNF conversion.



- 4 Empirical Evaluation
  - Relaxer Algorithm
  - Encoding Difficulty
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Domain	# Actions		# Ordering Constraints			
	KK	LCP	RX	MD	MR	LCP
Depots (14)	34.9	31.0	473.4	473.4	430.9	341.5
Driverlog (15)	27.5	26.5	332.6	332.6	326.9	297.3
Logistics (30)	78.1	77.4	1490.6	1490.6	1462.5	1470.4
TPP (5)	13.4	13.4	74.8	74.8	74.8	74.8
Rovers (18)	31.1	30.3	223.2	223.2	217.6	204.2
Zeno (16)	29.2	29.2	404.3	404.3	403.5	403.5
ALL (98)	44.3	43.2	685.7	685.7	669.0	651.6

Mean number of actions and ordering constraints. Number of actions for KK, MD, and MR are all equal.

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Four of the domains had problems with a reduction of actions.



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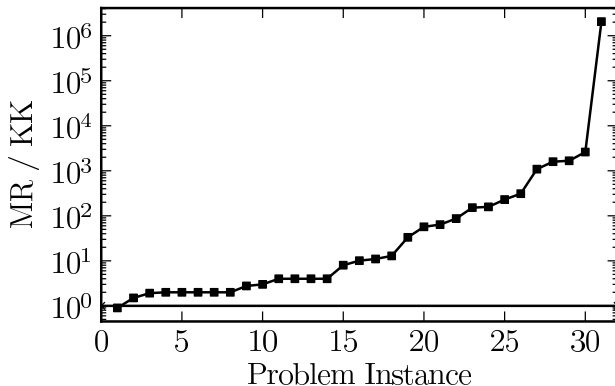
The Relaxer algorithm *always* computed the minimum deordering.

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Fewer actions may require an increase in ordering constraints.

- 4 Empirical Evaluation
  - Relaxer Algorithm
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# Reordering Flexibility



MR (resp. KK): The number of linearizations of the POP for the minimum reordering (resp. the POP generated by Relaxer).

- 1 Background
- 2 Least Commitment Criteria
- 3 Encoding
- 4 Empirical Evaluation
- 5 Conclusion

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- Proposed an extension to least commitment planning that includes the number of actions in a solution.
- Discovered that the Relaxer algorithm is extremely efficient at computing optimal deorderings.
- Found that greater flexibility can be achieved when using reorderings or the introduced least commitment criterion.

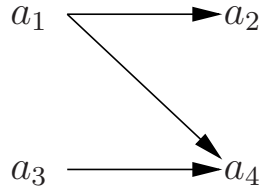
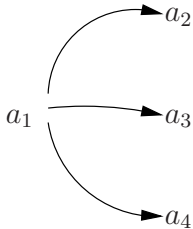
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- Use external reasoning for handling the transitive closure.
- Incorporate preferences into the optimization function.

<http://www.haz.ca/research/popgen/>

# Linearization Corner Case



- Boolean variables  $x_1, x_2, \dots$  that can be either True or False.
- Unary operator  $\neg$ , and binary operators  $\vee$  and  $\wedge$ .
- Well formed formula built by using variables,  $\neg$ ,  $\vee$ , and  $\wedge$ .
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## SAT Problem

Given a well formed formula, find a True / False setting to the variables such that the formula evaluates to True.



## MAXSAT

Given a CNF, find an assignment that satisfies as many of the clauses as possible.

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## Partial Weighted MAXSAT

Given a CNF with weights on *soft* clauses, find an assignment that satisfy all of the *hard* clauses and maximizes the sum of the weights on the satisfied clauses.

## Deordering

Let  $P = \langle \mathcal{A}, \mathcal{O} \rangle$  and  $Q = \langle \mathcal{A}', \mathcal{O}' \rangle$  be two POPs, and  $\Pi$  a planning problem.  $Q$  is a deordering of  $P$  wrt.  $\Pi$  iff  $P$  and  $Q$  are valid POPs for  $\Pi$ ,  $\mathcal{A} = \mathcal{A}'$ , and  $\mathcal{O}' \subseteq \mathcal{O}$ .

## Optimal Deordering

Let  $P = \langle \mathcal{A}, \mathcal{O} \rangle$  and  $Q = \langle \mathcal{A}', \mathcal{O}' \rangle$  be two POPs, and  $\Pi$  a planning problem.  $Q$  is a minimum deordering of  $P$  wrt.  $\Pi$  iff

- 1  $Q$  is a deordering of  $P$  wrt.  $\Pi$ , and
- 2 There is no deordering  $\langle \mathcal{A}'', \mathcal{O}'' \rangle$  of  $P$  wrt.  $\Pi$  s.t.  $|\mathcal{O}''| < |\mathcal{O}'|$

## Reordering

Let  $P = \langle \mathcal{A}, \mathcal{O} \rangle$  and  $Q = \langle \mathcal{A}', \mathcal{O}' \rangle$  be two POPs, and  $\Pi$  a planning problem.  $Q$  is a reordering of  $P$  wrt.  $\Pi$  iff  $P$  and  $Q$  are valid POPs for  $\Pi$ , and  $\mathcal{A} = \mathcal{A}'$ .

## Optimal Reordering

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- 1 Background
  - Propositional Planning
  - Partial Order Plans
  - Partial Weighted MAXSAT
- 2 Least Commitment Criteria
  - Deordering & Reordering
  - Least Commitment POP
- 3 Encoding
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